M.Sc.

[Mathematics with Computer Science]
[Applied Mathematics]
[Mathematics]

Syllabus

(As per UGC CBCS w.e.f. 2016–17)

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

Hyderabad

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY

(Choice Based Credit System)

(w.e.f. The Academic year 2016-2017)

M.Sc. (Mathematics with Computer Science)

SEMESTER – I

S. No.	Sub. Code.	Subject	Hrs./ Week	Internal Assessment	Semester Exam	Total Marks	Credits
1. Core	MCS101	Algebra	4	20	80	100	4
2. Core	MCS102	Analysis	4	20	80	100	4
3. Core	MCS103	Operating Systems	4	20	80	100	4
4. Core	MCS104	OOPs through Java	4	20	80	100	4
5. Practical	MCS151	Algebra	4	--	50	50	2
6. Practical	MCS152	Analysis	4	--	50	50	2
7. Practical	MCS153	Operating System	4	--	50	50	2
8. Practical	MCS154	OOPs through Java	4	--	50	50	2
		Total	**32**			**600**	**24**

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. (Mathematics with computer science)

Algebra

MCS 101 Paper I Semester I

Unit I

Automaphisms- Conjugacy and G-sets- Normal series solvable groups- Nilpotent groups. (Pages 104 to 128 of [1])

Unit II

Structure theorems of groups: Direct product- Finitly generated abelian groups- Invariants of a finite abelian group- Sylow's theorems- Groups of orders p^2,pq . (Pages 138 to 155)

Unit III

Ideals and homomsphism- Sum and direct sum of ideals, Maximal and prime ideals- Nilpotent and nil ideals- Zorn's lemma (Pages 179 to 211).

Unit-IV

Unique factorization domains - Principal ideal domains- Euclidean domains- Polynomial rings over UFD- Rings of traction.(Pages 212 to 228)

Text Books:

[1] Basic Abstract Algebra by P.B. Bhattacharya, S.K. Jain and S.R. Nagpanl.

Reference:

[1] Topics in Algebra by I.N. Herstein.

M.Sc. (Mathematics with Computer Science)

Algebra

MCS 151 Paper I Semester I

Practical Questions

1. A finite group G having more than two elements and with the condition that $x^2 \neq e$ for some $x \in G$ must have nontrivial automorphism.
2. (i) Let G be a group Define $a * x = ax,\ a, x \in G$ then the set G is a G-set
   (ii) Let G be a group Define $a * x = axa^{-1}\ a, x \in G$ then G is a G-set.
3. An abelian group G has a composition series if and only if G is finite
4. Find the number of different necklaces with p beads p prime where the beads can have any of n different colours
5. If G is a finite cyclic group of order n then the order of Aut G, the group of automorphisms of G, is $\phi(n)$, where ϕ is Euler's function.
6. If each element $\neq e$ of a finite group G is of order2 then $|G| = 2^n$ and $G \approx C_1 \times C_2 \times \ldots\ldots \times C_n$ where C_i are cyclic and $|C_i| = 2$.
7. (i) Show that the group $\frac{Z}{<10>}$ is a direct sum of $H = \{\bar{0}\ \bar{5}\}$ *and* $K = \{\bar{0}\ \bar{2}\ \bar{4}\ \bar{6}\ \bar{8}\}$
   (ii) Show that the group $\left(\frac{z}{\langle 4 \rangle}, +\right)$ cannot be written as the direct sum of two Subgroups of order 2.
8. (i) Find the non-isomorphic abelian groups of order 360
   (ii) If a group of order p^n contains exactly one sub group each of orders $p, p^2, \_\ \_\_ P^{n-1}$ then it is cyclic.
9. Prove that there are no simple groups of orders 63, 56, and 36
10. Let G be a group of order 108. Show that there exists a normal subgroup of order 27 or 9.
11. (i) Let **R** be a commutative Ring with unity. Suppose R has no nontrivial ideals .Prove that R is a field. (ii) Find all ideals in Z and in $\frac{Z}{\langle 10 \rangle}$
12. (i) The only Homomorphism from the ring of integers Z to Z are the identity and Zero Mappings.
    (ii) Show that any nonzero homomorphism of a field F into a ring R is one-one.
13. For any tow ideals A and B in a Ring R (i) $\frac{A+B}{B} \approx \frac{A}{A \cap B}$
    (ii) $\frac{A+B}{A \cap B} \approx \frac{A+B}{A} \times \frac{A+B}{B} \approx \frac{B}{A \cap B} \times \frac{A}{A \cap B}$
    In particular if $R = A + B$ then $\frac{R}{A \cap B} \approx \frac{R}{A} \times \frac{R}{B}$.
14. Let R be a commutative ring with unity in which each ideal is prime then R is a field.

15. Let R be a Boolean ring then each prime ideal $P \neq R$ is maximal.
16. The commutative integral domain $R = \{a + b\sqrt{-5} / a, b \in Z\}$ is not a UFD.
17. (i) The ring of integers Z is a Euclidean domain
    (ii) The Ring of Gausion Integers $R = \{m + n\sqrt{-1} / m, n \in Z\}$ is a Euclidean domain
18. (i) Prove that $2 + \sqrt{-5}$ is irreducible but not prime in $Z(\sqrt{-5})$
    (ii) Show that $1 + 2\sqrt{-5}$ *and* 3 are relatively prime in $Z(\sqrt{-5})$
19. Let R be a Euclidean domain. Prove the following
    (i) If $b \neq 0$ *then* $\phi(a) < \phi(b)$
    (ii) If a and b are associates then $\phi(a) = \phi(b)$
    (iii) If a/b *and* $\phi(a) = \phi(b)$ *then* a and b are associates
20. Prove that every nonzero prime ideal in a Euclidean domain is maximal.

DEPARTMENT OF MATHEMATICS
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M.Sc. (Mathematics with Computer Science)

MCS 102 Semester I

Analysis

Paper – II

Unit I

Metric spaces – Compact sets – Perfect sets – Connected sets.

Unit II

Limits of functions- Continuous functions- Continuity and compactness Continuity and connectedness- Discontinuities – Monotone functions.

Unit III

Rieman- Steiltjes integral- Definition and Existence of the Integral- Properties of the integral- Integration of vector valued functions- Rectifiable curves.

Unit-IV

Sequences and series of functions: Uniform convergence- Uniform convergence and continuity- Uniform convergence and integration- Uniform convergence and differentiation- Approximation of a continuous function by a sequence of polynomials.

Text Book

[1] Principles of Mathematical Analysis (3e) by Walter Rudin

M.Sc. (Mathematics with Computer Science)

Analysis

MCS 152 Paper –II Semester -I

Practical Questions

1. Construct a bounded set of real numbers with exactly three limit points
2. Suppose E^1 is the set of all limit points of E. Prove that E^1 is closed also prove that E and E have the same limit points.
3. Let E^0 demote the set of all interior points of a set E. Prove that E^0 is the largest open set contained in E Also prove that E is open if and only if $E = E^0$
4. Let Xbe an infinite set.For $p \in X$, $q \in X$ define

$$d(p,q) = \begin{cases} 1 \text{ if } p \neq q \\ 0 \text{ if } p = q \end{cases}$$

5. Prove that this is a metric, which subsets of the resulting metric space are open, which are closed? Which are compact?
6. i) If A and B are disjoint closed sets in some metric space X, prove that they are separated
   ii) Prove the same for disjoint open sets
   iii)Fixa $p \in X$ and $\delta > o$, Let A = $\{ q \in X : d(p,q) < \delta \}$ and $B = \{q \in X : d(p,q) > \delta \}$ prove that A and B are separated.
7. i) Suppose f is a real function on R which satisfies $\lim_{h \to o}[f(x+h) - f(x-h) = o]$ for every $x \in R$ Does this imply that f is continuous? Explain
   ii) Let f be a continuous real function on a metric space X, let Z(f) = $\{p \in X : f(p) = 0\}$. Prove that z (f) is closed.
8. If f is a continuous mapping of a metric space X into a metric space Y. Prove that

   $f(\overline{E}) \subset \overline{f(E)}$ for every set $E \subset X$

9. Let f and g be continuous mapping of a metric space X into a metric space Y Let E be a dense subset of X. Prove that
   i) f(E) is dense in f(X)
   ii) If $g(p) = f(p)$ $\forall p \in E$, Prove that $g(p) = f(p)$ $\forall p \in X$
10. Suppose f is a uniformly continuous mapping of a metric space X into a metric space Y and $\{ X_m\}$ is a Couchy sequence in X prove that $\{f(X_m)\}$ is Cauchy sequence in Y
11. Let I = [0 , 1] be the closed unit interval, suppose f is a continuous mapping of f into I. Prove that f(x) = x for at least one x
12. Suppose α increases on [a , b] , a <x_o<b, α is continuous at x_0, $f(x_0) = 1$ and f(x) =0 if x $\neq x_o$ .Prove that $f \in R(\alpha)$ and $\int_a^b f \, d\alpha = 0$
13. Suppose $f \geq 0$ and f is continuous on [a, b] and $\int_a^b f(x)dx = 0$, Prove that f(x) =0 $\forall$ x€ [a, b]

14. If f(x) = 1or 0 according as x is rational or not .Prove that f $\notin R$ on [a, b] for any a, b €R with a<b. Also prove that f $\notin$ R(α) on [a , b] with respect to any monotonically increasing function α on [a, b]
15. Suppose f is a bounded real function on [a, b] and f^2€R on [a, b]. Does it follow that $f \in R$? Does the answer change if we assume that f^3€R?
16. Suppose γ_1 *and* γ_2 are the curves in the complex plane defined on [0, 2π] by $\gamma_1(t) = e^{it}$, $\gamma_2(t) = e^{2it}$

    Show that the two curves have the same range

    Also Show that γ_1 *and* γ_2 are rectifiable and find the curve length of γ_1 *and* γ_2
17. Discuss the uniform conversance of the sequence of functions $\{f_n\}$ where

$$f_n(x) = \frac{\sin nx}{\sqrt{n}} \quad \text{x real, n = 1,2,3....}$$

18. Give an example of a series of continuous functions whose sum function may be discontinuous.
19. Discuss the uniform conversance of the sequence

$$f_n(x)=\frac{1}{1+nx} \; x > 0, \; n = 1,2,3...$$

20. Give an example of a sequence of functions such that

$$\lim \int f_n \neq \int \lim f_n$$

21. Prove that a sequence {fn} converse to f with respect to the metric of C(x) if and only if fn$\rightarrow$f uniformly on X

DEPARTMENT OF MATHEMATICS
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M.Sc. (Mathematics with computer science)
Operating Systems

MCS 103 Paper III Semester I

Unit I

Introduction – Mainframe systems –Desktop Systems - Multiprocessor Systems - Distributed Systems Clustered Systems – Real Time Systems – Handheld Systems - Hardware Protection – System Components – Operating System Services – System Calls – System Programs – Process Concept – Process Scheduling – Operations on Processes – Cooperating Processes – Inter – process Communication.

Unit II

Threads – Overview –Threading issues – CPU Scheduling – Basic Concepts – Scheduling Criteria – Scheduling Algorithms – Multiple Processor Scheduling – Real Time Scheduling – The Critical Section Problem Synchronization Hardware – Semaphores – Classic problems of Synchronization – Critical regions – monitors.

Unit III

System Model – Deadlock Characterization – Methods for handling Deadlocks – Deadlock Prevention – Deadlock avoidance –Deadlock detection – Recovery from Deadlocks - Storage Management – Swapping – Contiguous Memory Allocation - Paging – Segmentation – Segmentation with paging.

Unit IV

Virtual Memory – Demand Paging – Process Creation – Page Replacement – Allocation of frames – Thrashing – file Concept – Access Methods – Directory Structure – File System Mounting File Sharing – Protection – File System Structure – File System Implementation – Directory Implementation – Allocation Methods- Free space Management – Kernel I/O Subsystems –Disk Structure – Disk Scheduling – Disk Management – Swap Space Management. Case study: The Linux System, Windows.

Reference Books:

1. Abraham Silberschatz, Peter Baer Galvin and Greg Gagne, “Operating System Concepts”, Sixth Edition, John Wiley & Sons (ASIA) Pvt. Ltd.
2. Harvey M. Deitel, “Operating Systems”, Second Edition, Pearson Education Pvt. Ltd.
3. Andrew S. Tanenbaum, “Modern Operating Systems”, Prentice Halla of India Pvt. Ltd.
4. William Stallings, “Operating System”, Prentice Hall if India, 4^{th} Edition, 2003
5. Promod Chnadra P.Bhatt, “An Introduction to Operating Systems, Concepts and Practice”, PHI.

M.Sc. (Mathematics with Computer Science)

Operating Systems

MCS 153 Paper–III Semester – I

Practical Questions

(Implement the following on LINUX platform. Use C for high level language implementation)

1. Shell programming
   - Command syntax
   - Write simple functions
   - Basic tests
2. Shell programming
   - Loops
   - Patterns
   - Expansions
   - Substitutions
3. Write programs using the following system calls of UNIX operating system: fork, exec, getpid, exit, wait, close, stat, opendir, readdir.
4. Write programs using the I/O system calls of UNIX operating system (open, read, write, etc)
5. Write C programs to simulate UNIX commands like ls, grep, etc.
6. Given the list of processes, their CPU burst times and arrival times, display/print the Gantt chart for FCFS and SJF. For each of the scheduling policies, compute and print the average waiting time and average turnaround time
7. Given the list of processes, their CPU burst times and arrival times, display/print the Gantt chart for Priority and Round robin. For each of the scheduling policies, compute and print the average waiting time and average turnaround time
8. Implement the Producer – Consumer problem using semaphores.
9. Implement some memory management schemes – I
10. Implement some memory management schemes – II

EXAMPLE FOR EXPT 9 &10:

Free space is maintained as a linked list of nodes with each node having the starting byte address and the ending byte address of a free block. Each memory request consists of the process-id and the amount of storage space required in bytes. Allocated memory space is again maintained as a linked list of nodes with each node having the process-id, starting byte address and the ending byte address of the allocated space.

When a process finishes (taken as input) the appropriate node from the allocated list should be deleted and this free disk space should be added to the free space list. [Care should be taken to merge contiguous free blocks into one single block. This results in deleting more than one node from the free space list and changing the start and end address in the appropriate node]. For allocation use first fit, worst fit and best fit.

DEPARTMENT OF MATHEMATICS
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M.Sc. (Mathematics with Computer Science)
OOPs through Java

MCS 104 Paper IV Semester I

Unit I

Introduction to Core Java – Class and Object, Object Oriented concepts with respect to Java, Interfaces, Packages and Exception Handling, Applets, Abstract Window Toolkit and Swing Components and Graphics, Containers, Frames and Panels, Layout Managers Border layout, Flow layout Grid layout, Card layout, AWT all components, Swing & Its Features, JApplet, Icons & Labels Button & Label, Text Field & Toggle Buttons, checkboxes, Radio buttons, Combo Box & Lists, Scroll panes, Trees, Tables, Menu Bars & Menus, Tool Bars, Dialog Boxes, File Dialog, Progress Bar, Choosers.

Unit II

Multithreading and I/O – Multithreading concepts, Thread Lifecycle, Creating multithreaded application, Thread priorities, Thread synchronization. Java Input Output – Java IO package, Byte/Character Stream, Buffered reader/writer, File reader/writer, Print writer, File Sequential/Random.

Unit III

Java Database Connectivity (JDBC) – Introduction to JDBC, Types of JDBC Connectivity, Types of statement objects (Statement, Prepared Statement and Callable Statement), Types of result set, Result Set Metadata, Inserting and updating records, JDBC and AWT Connection pooling.

Unit IV

RMI and Servlet – Introduction & Architecture of RMI, Java RMI classes and interfaces, Writing simple RMI application, Parameter passing in remote methods (marshalling and un-marshalling). Servlet Overview & Architecture, Setting up Apache Tomcat Server, Handling HTTP Get Request, Handling HTTP Get Request Containing Data Handling HTTP Post Request.

Suggested readings:

1. Herbert Schildt, “Java: The Complete Reference”, Tata McGraw-Hill
2. DeitelandDeitel."Java: How to Program", Addison-Wesley Press, Reading, Mass
3. David Flanagan, "Java in a Nutshell (Java2.1)", 2nd Ed., O'Reilly and Associates Publishing

M.Sc. (Mathematics with Computer Science)

MCS 154 | OOPs through Java | Semester – II

Paper – IV

Practical Questions

1. Simple structure of Java program
2. Write the Interfaces
3. Creating Packages
4. Program based on Exception Handling
5. Program based on Applets
6. Program based on Designing of Frames
7. Program based on Inserting components on frame
8. Program to demonstrate Layouts
9. Program based on Action Listener
10. Programs based on Menus and Dialog boxes
11. Program based on Multithreading
12. Program based on I/O
13. Program based on JDBC connectivity (insert, delete, update operations)
14. Program based on RMI
15. Program based on Servlet

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY

(Choice Based Credit System)

(w.e.f. The Academic year 2016-2017)

M.Sc. (Mathematics with Computer Science)

SEMESTER – II

S. No.	Sub. Code.	Subject	Hrs./ Week	Internal Assessment	Semester Exam	Total Marks	Credits
1. Core	MCS201	Complex Analysis	4	20	80	100	4
2. Core	MCS202	Functional Analysis	4	20	80	100	4
3. Core	MCS203	Software Engineering	4	20	80	100	4
4. Core	MCS204	Programming using Python	4	20	80	100	4
5. Practical	MCS251	Complex Analysis	4	--	50	50	2
6. Practical	MCS252	Functional Analysis	4	--	50	50	2
7. Practical	MCS253	Software Engineering	4	--	50	50	2
8. Practical	MCS254	Programming using Python	4	--	50	50	2
		Total	**32**			**600**	**24**

DEPARTMENT OF MATHEMATICS
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M.Sc. (Mathematics with Computer Science)

MCS201 Semester II

Complex Analysis

Paper – I

Unit I

Regions in the Complex Plane – Functions of a Complex Variable – Mappings – Mappings by the Exponential Function – Limits – Limits involving the Point at Infinity – Continuity – Derivatives – Cauchy–Riemann Equations – Sufficient Conditions for Differentiability – Analytic Functions –Harmonic Functions – Uniquely Determined Analytic Functions – Reflection Principle – The Exponential Function – The Logarithmic Function – Some Identities Involving Logarithms - Complex Exponents – Trigonometric Functions – Hyperbolic Functions.

Unit II

Derivatives of Functions w(t) – Definite Integrals of Functions w(t) – Contours – Contour Integrals – Some Examples – Examples with Branch Cuts – Upper Bounds for Moduli of Contour Integrals –Anti derivatives – Cauchy–Goursat Theorem – Simply Connected Domains – Multiply Connected Domains – Cauchy Integral Formula – An Extension of the Cauchy Integral Formula – Liouville's Theorem and the Fundamental Theorem of Algebra – Maximum Modulus Principle.

Unit III

Convergence of Sequences – Convergence of Series – Taylor Series – Laurent Series – Absolute and Uniform Convergence of Power Series – Continuity of Sums of Power Series – Integration and Differentiation of Power Series – Uniqueness of Series Representations- Isolated Singular Points – Residues – Cauchy's Residue Theorem – Residue at Infinity – The Three Types of Isolated Singular Points – Residues at Poles – Examples – Zeros of Analytic Functions – Zeros and Poles – Behavior of Functions Near Isolated Singular Points.

Unit IV

Evaluation of Improper Integrals – Improper Integrals from Fourier Analysis – Jordan's Lemma – Indented Paths – Definite Integrals Involving Sines and Cosines – Argument Principle – Rouché's Theorem – Linear Transformations – The Transformation $w = 1/z$ – Mappings by $1/z$ – Linear Fractional Transformations – An Implicit Form – Mappings of the Upper Half Plane.

Text Book:

1. James Ward Brown, Ruel V Churchill, Complex Variables with Applications

M.Sc. (Mathematics with Computer Science)

Complex Analysis

MCS251 | Paper – I | Semester – II

Practical Questions

1

In each case, determine the singular points of the function and state why the function is analytic everywhere except at those points:

(a) $f(z) = \dfrac{2z+1}{z(z^2+1)}$; (b) $f(z) = \dfrac{z^3+i}{z^2-3z+2}$; (c) $f(z) = \dfrac{z^2+1}{(z+2)(z^2+2z+2)}$.

2

Show that $u(x, y)$ is harmonic in some domain and find a harmonic conjugate $v(x, y)$ when

(a) $u(x, y) = 2x(1 - y)$; (b) $u(x, y) = 2x - x^3 + 3xy^2$;
(c) $u(x, y) = \sinh x \sin y$; (d) $u(x, y) = y/(x^2 + y^2)$.

3

Find all values of z such that

(a) $e^z = -2$; (b) $e^z = 1 + \sqrt{3}i$; (c) $\exp(2z - 1) = 1$.

4

Let the function $f(z) = u(x, y) + iv(x, y)$ be analytic in some domain D. State why the functions

$$U(x, y) = e^{u(x,y)} \cos v(x, y), \quad V(x, y) = e^{u(x,y)} \sin v(x, y)$$

are harmonic in D and why $V(x, y)$ is, in fact, a harmonic conjugate of $U(x, y)$.

5

Show that

(a) $(1 + i)^i = \exp\left(-\dfrac{\pi}{4} + 2n\pi\right) \exp\left(i\dfrac{\ln 2}{2}\right) \quad (n = 0, \pm 1, \pm 2, \ldots)$;

(b) $(-1)^{1/\pi} = e^{(2n+1)i} \quad (n = 0, \pm 1, \pm 2, \ldots)$.

6

Let C denote the line segment from $z = i$ to $z = 1$. By observing that of all the points on that line segment, the midpoint is the closest to the origin, show that

$$\left|\int_C \frac{dz}{z^4}\right| \le 4\sqrt{2}$$

without evaluating the integral.

7

Show that if C is the boundary of the triangle with vertices at the points $0, 3i$, and -4, oriented in the counterclockwise direction (see Fig. 48), then

$$\left|\int_C (e^z - \bar{z})\, dz\right| \le 60.$$

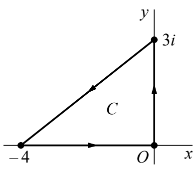


FIGURE 48

8

Let C be the unit circle $z = e^{i\theta} (-\pi \le \theta \le \pi)$. First show that for any real constant a,

$$\int_C \frac{e^{az}}{z}\,dz = 2\pi i.$$

Then write this integral in terms of θ to derive the integration formula

$$\int_0^{\pi} e^{a\cos\theta}\cos(a\sin\theta)\,d\theta = \pi.$$

9

Find the value of the integral of $g(z)$ around the circle $|z - i| = 2$ in the positive sense when

(a) $g(z) = \dfrac{1}{z^2 + 4}$; *(b)* $g(z) = \dfrac{1}{(z^2 + 4)^2}$.

10

Show that for R sufficiently large, the polynomial $P(z)$ in Theorem 2, Sec. 53, satisfies the inequality

$$|P(z)| < 2|a_n||z|^n \quad \text{whenever} \quad |z| \ge R.$$

11

Obtain the Maclaurin series representation

$$z\cosh(z^2) = \sum_{n=0}^{\infty} \frac{z^{4n+1}}{(2n)!} \qquad (|z| < \infty).$$

12

Obtain the Taylor series

$$e^z = e\sum_{n=0}^{\infty} \frac{(z-1)^n}{n!} \qquad (|z-1| < \infty)$$

for the function $f(z) = e^z$ by

13

In each case, show that any singular point of the function is a pole. Determine the order m of each pole, and find the corresponding residue B.

(a) $\dfrac{z^2 + 2}{z - 1}$; *(b)* $\left(\dfrac{z}{2z + 1}\right)^3$; *(c)* $\dfrac{\exp z}{z^2 + \pi^2}$.

14

Evaluate the integral

$$\int_C \frac{\cosh \pi z}{z(z^2 + 1)}\,dz$$

when C is the circle $|z| = 2$, described in the positive sense.

15

Show that

(a) $\displaystyle \operatorname*{Res}_{z=\pi i} \frac{z - \sinh z}{z^2 \sinh z} = \frac{i}{\pi}$;

(b) $\displaystyle \operatorname*{Res}_{z=\pi i} \frac{\exp(zt)}{\sinh z} + \operatorname*{Res}_{z=-\pi i} \frac{\exp(zt)}{\sinh z} = -2\cos(\pi t)$

16

Evaluate

$$\int_{-\infty}^{\infty} \frac{\cos x\,dx}{(x^2 + a^2)(x^2 + b^2)} \quad (a > b > 0).$$

17

Derive the integration formula

$$\int_0^{\infty} \frac{\cos(ax) - \cos(bx)}{x^2}\,dx = \frac{\pi}{2}(b - a) \qquad (a \geq 0, b \geq 0).$$

Then, with the aid of the trigonometric identity $1 - \cos(2x) = 2\sin^2 x$, point out how it follows that

$$\int_0^{\infty} \frac{\sin^2 x}{x^2}\,dx = \frac{\pi}{2}.$$

18

Evaluate

$$\int_0^{\pi} \frac{\cos 2\theta\, d\theta}{1 - 2a\cos\theta + a^2} \quad (-1 < a < 1)$$

19

Suppose that a function f is analytic inside and on a positively oriented simple closed contour C and that it has no zeros on C. Show that if f has n zeros z_k $(k = 1, 2, \ldots, n)$ inside C, where each z_k is of multiplicity m_k, then

$$\int_C \frac{zf'(z)}{f(z)}\,dz = 2\pi i \sum_{k=1}^{n} m_k z_k.$$

20

Determine the number of zeros, counting multiplicities, of the polynomial

(a) $z^4 + 3z^3 + 6$; (b) $z^4 - 2z^3 + 9z^2 + z - 1$; (c) $z^5 + 3z^3 + z^2 + 1$

inside the circle $|z| = 2$.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Mathematics with Computer Science)

MCS202 Semester II

Functional Analysis

Paper – II

Unit I

Normed Linear Spaces: Definitions and Elementary Properties, Subspace, Closed Subspace, Finite Dimensional Normed Linear Spaces and Subspaces, Quotient Spaces, Completion of Normed Spaces.

Unit II

Hilbert Spaces: Inner Product Space, Hilbert Space, Cauchy-Bunyakovsky-Schwartz Inequality, Parallelogram Law, Orthogonality, Orthogonal Projection Theorem, Orthogonal Complements, Direct Sum, Complete Orthonormal System, Isomorphism between Separable Hilbert Spaces.

Unit III

Linear Operators: Linear Operators in Normed Linear Spaces, Linear Functionals, The Space of Bounded Linear Operators, Uniform Boundedness Principle, Hahn-Banach Theorem, Hahn-Banach Theorem for Complex Vector and Normed Linear Space, The General Form of Linear Functionals in Hilbert Spaces.

Unit IV

Fundamental Theorems for Banach Spaces And Adjoint Operators In Hilbert Spaces: Closed Graph Theorem, Open Mapping Theorem, Bounded Inverse Theorem, Adjoint Operators, Self-Adjoint Operators, Quadratic Form, Unitary Operators, Projection Operators.

Text Book:

1. A First Course in Functional Analysis by Rabindranath Sen, Anthem Press.

Reference:

1. Introductory Functional Analysis by E.Kreyzig, John Wilely and Sons, New York,
2. Functional Analysis, by B.V. Limaye 2nd Edition.
3. Introduction to Topology and Modern Analysis by G.F.Simmons. Mc.Graw-Hill

M.Sc. (Mathematics with Computer Science)

Functional Analysis

MCS252 Paper – II Semester – II

Practical Questions

1. Let ρ be the matric induce by a norm on a linear space $E \neq \phi$. If ρ_1 is defined by

$$\rho_1(x,y) = \begin{cases} 0 & x = y \\ 1 + \rho(x,y) & x \neq y \end{cases}$$

then prove that ρ_1 can't be obtain from a norm on E.

2. (a). Show that the closure $\bar{X}$ of a subspace X of a normed linear space E is again a subspace of E.
(b). Prove that the intersection of an arbitrary collection of non-empty closed subspaces of the normed linear space E is a closed subspace of E.

3. Let E_1 be a closed subspace and E_2 be a finite dimensional subspace of a normed linear space E. Then show that $E_1 + E_2$ is closed in E.

4. Show that a finite dimensional normed linear space is separable.

5. Show that equivalent norms on a vector space E induces the same topology on E.

6. Let C be a convex set in a Hilbert space H, and $d = inf\{\|x\|, x \in C\}$.If $\{x_n\}$ is a sequence in C such that $\lim_{x\to\infty}\|x_n\| = d$, show that $\{x_n\}$ is a Cauchy sequence.

7. Show that if M and N are closed subspaces of a Hilbert space H, then $M + N$ is closed provided $x \perp y$ for all $x \in M$ and $y \in N$.

8. Let $\{a_1, a_2, ...a_n\}$ be an orthogonal set in a Hilbert space H, and $\alpha_1, \alpha_2, ...\alpha_n$ be scalars such that their absolute values are respectively 1. Show that $\|\alpha_1 a_1 + ... + \alpha_n a_n\| = \|a_1 + a_2 + ... + a_n\|$.

9. Let H be a Hilbert space, $M \subseteq H$ a convex subset, and x_n a sequence in M such that $\|x_n\| \to d$ as $n \to \infty$ where $d = \inf_{x\in M} \|x\|$.Show that $\{x_n\}$ converges in H.

10. Let $x_1, x_2, ...x_n$ satisfy $x_i \neq 0$ and $x_i \perp x_j$ if $i \neq j$, $i, j = 1, 2, ...n$. Show that the $x_i's$ are linearly independent and extend the Pythagorean theorem from 2 to n dimensions.

11. Let E be a linear space over a scalar field $\mathbb{R}(or\mathbb{C})$. Prove that the space of continuous linear operators mapping E into itself is a ring.

12. Prove that every linear operator on a normed space is continuous iff bounded.

13. Give an example of an linear operator which is not bounded. Explain.

14. Let $x_0 \neq 0$ be a fixed element in a normed linear space E. Then prove that there exists a linear functional $f(x)$, defined on the entire space E, such that $\|f\| = 1$ and $f(x_0) = \|x_0\|$.

15. Let L be a closed linear subspace of a normed linear space E, and x_0 be a vector not in L. If d is the distance from x_0 to L, show that there exists a functional $f_0 \in E^*$ such that $f_0(L) = 0$, $f_0(x_0) = 1$ and $\|f_0\| = \frac{1}{d}$.

16. Given that E is a Banach space, $\mathcal{D}(T) \subseteq E$ is closed, and the linear operator T is bounded, show that T is closed.

17. Prove that for the projections P_1 and P_2 to be orthogonal, it is necessary and sufficient that the corresponding subspace L_1 and L_2 are orthogonal.

18. Give an example of a normal operator which is neither self-adjoint nor unitary. Explain.

19. Let $\|.\|$ and $\|.\|'$ be norms on a linear space E. Then prove that the norm $\|.\|$ is stronger than $\|.\|'$ if and only if there is some $\alpha > 0$ such that $\|x\| \leq \alpha\|x\|'$ for all $x \in E$.

20. Prove that P is a self-adjoint operator with its norm equal to one and P satisfies $P^2 = P$.

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. (Mathematics with Computer Science)

Software Engineering

MCS 203 Paper III Semester II

Unit I

Introduction to software Engineering – Project size and its categories – planning a software project – software developing life cycle – planning and organizational Structure.

Unit II

Software cost estimation, least factor – cost estimation techniques – maintenance cost estimation – Software Requirement Specifications – formal specification techniques.

Unit III

Software Design – Fundamental Design concepts and relations of Modularization – Module design techniques – detailed design consideration – Implementation issues – Structures coding techniques – coding style – standards and guidelines – Documentation – Verification and Validation techniques – Quality Assurance – Walk through and inspection – Testing – format verification.

Unit IV

Software tools – Overview of CASE – Software reliability – Software errors – Faculty – Repairs and availability – Software maintenance – Management aspects of maintenance – Maintenance tools and techniques.

Text book:

1. R. Fairly – Software Engineering, Mc. Graw Hills Publishing Co.
2. Pressman – Software Engineering, Mc. Graw Hills Publishing Co.

M.Sc. (Mathematics with Computer Science)

MCS 253 Software Engineering Semester – II

Paper – III

Practical Questions

1. Study of case tool
   Requirements
2. Implementation of requirements engineering activities such as elicitation, validation, management using case tools
   Analysis and Design
   Implementation of Analysis and design using case tools
3. Study and usage of software project management tools such cost estimates and scheduling
4. Documentation generators – Study and practice of Documentation generators
5. Data Modelling using automated tools
6. Practice reverse engineering and re-engineering using tools
7. Exposure towards test plan generators, test case generators, test coverage and software metrics.
8. Meta modelling and software life cycle management.
   Case Studies:
9. Structure charts, Data Flow Diagrams, Decision tables and ER diagrams for Banking System
10. Structure charts, Data Flow Diagrams, Decision tables and ER diagrams for Railway Reservation System
11. Structure charts, Data Flow Diagrams, Decision tables and ER diagrams for Food Ordering System
12. Structure charts, Data Flow Diagrams, Decision tables and ER diagrams for Inventory System

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Mathematics with Computer Science)
Programming using Python

MCS 204 Paper IV Semester II

Unit I

Introduction to Programming Languages – What is program and programming paradigms, Programming languages – their classification and characteristics, language translators and language translation activities, Use of Algorithms/Flow Charts for problem solving.
Building Blocks of Program – Data, Data Types, Data Binding, Variables, Constants, Declaration, Operations on Data such as assignment, arithmetic, relational, logical operations, dry run, Evaluating efficiency of algorithms in terms of number of operations and variables.
Introduction to Python Programming – Features, basic syntax, Writing and executing simple program, Basic Data Types such as numbers, strings etc... Declaring variables, Performing assignments, arithmetic operations, Simple input-output.

Unit II

Sequence Control – Precedence of operators, Type conversion, Conditional Statements – if, if-else, nested if-else, looping – for, while, nested loops, Control statements – Terminating loops, skipping specific conditions, String Manipulation – declaring strings, string functions, Manipulating Collections Lists, Tuples, Dictionaries – Concept of dictionary, techniques to create, update & delete dictionary items.
Functions – Defining a function, calling a function, Advantages of functions, types of functions, function parameters, Formal parameters, Actual parameters, anonymous functions, global and local variables, Modules – Importing module, Creating & exploring modules, Math module, Random module, Time module.

Unit III

GUI Programming in Python (using Tkinter / wxPython / Qt) – What is GUI, Advantages of GUI, Introduction to GUI library, Layout management, Events and bindings, Font, Colors, drawing on canvas (line, oval, rectangle, etc.) Widget such as: Frame, Label, Button, Check button, Entry, List box, Message, Radio button, Text, Spin box etc…
Python File Input-Output – Opening and closing file, Various types of file modes, reading and writing to files, manipulating directories.

Unit IV

Exception Handling – What is exception, various keywords to handle exception such try, catch, except, else, finally, raise, Regular Expressions – Concept of regular expression, various types of regular expressions, using match functions. Database connectivity in Python – Installing MySQL connector, accessing connector module, using connect, cursor, execute & close functions, reading single & multiple results of query execution, executing different types of statements, executing transactions, understanding exceptions in database connectivity
Algorithm, Searching and Sorting – Searching and sorting techniques, Efficiency of algorithms.

Text Books:

1. Charles Dierbach, Introduction to Computer Science using Python, Wiley,2013
2. James Payne , Beginning Python: Using Python 2.6 and Python 3, Wiley India,2010
3. Paul Gries , Jennifer Campbell, Jason Montojo, Practical Programming: An Introduction to Computer Science Using Python 3, Pragmatic Bookshelf, 2/E2014
4. James Payne , Beginning Python: Using Python 2.6 and Python 3, Wiley India,2010
5. Adesh Pandey, Programming Languages – Principles and Paradigms, Narosa,2008
6. Lukaszewski, MySQL for Python: Database Access Made Easy, Pact Publisher, 2010

M.Sc. (Mathematics with Computer Science)

MCS 254 Programming using Python Semester – II

Paper – IV

Practical Questions

1. Using the Operating system (logging, creating – deleting folders, creating-deleting files, using editors etc.)
2. Installing python and setting up environment. Simple statements like printing the names, numbers, mathematical calculations, etc.
3. Simple programs containing variable declaration and arithmetic operations
4. Programs based on conditional constructs
5. Programs based on loops
6. Programs related to string manipulation
7. Programs related to Lists, Tuples
8. Programs related to dictionaries
9. Programs related to functions &modules
10. Programs to read & write file.
11. Program to demonstrate exception handling
12. Program to demonstrate the use of regular expressions
13. Program to draw shapes
14. Program to show GUI controls and processing–I
15. Program to show GUI controls and processing–II
16. Program to show database connectivity
17. Programs to do searching and sorting

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY

(Choice Based Credit System)

(w.e.f. The Academic year 2017–2017)

M.Sc. (Mathematics with Computer Science)

SEMESTER – III

S. No.	Sub. Code.	Subject	Hrs./ Week	Internal Assessment	Semester Exam	Total Marks	Credits
1. Core	MCS301	Computer Networks	4	20	80	100	4
2 Core	MCS302	Mathematical Methods	4	20	80	100	4
3. Elective	MCS303A	Advanced RDBMS	4	20	80	100	4
	MCS303B	Design and Analysis of Algorithms					
	MCS303C	Object Oriented Analysis and Design					
	MCS303D	Robotics and Artificial Intelligence					
4. Elective	MCS304A	Elementary Number Theory	4	20	80	100	4
	MCS304B	Integral Equations and Calculus of Variations					
	MCS304C	Operations Research					
5. Practical	MCS351	Computer Networks	4	--	50	50	2
6. Practical	MCS352	Mathematical Methods	4	--	50	50	2
7. Practical	MCS353A	Advanced RDBMS	4	--	50	50	2
	MCS353B	Design and Analysis of Algorithms					
	MCS353C	Object Oriented Analysis and Design					
	MCS353D	Robotics and Artificial Intelligence					
8. Practical	MCS354A	Elementary Number Theory	4	--	50	50	2
	MCS354B	Integral Equations & Calculus of Variations					
	MCS354C	Operations Research					
		Total	**32**			**600**	**24**

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. (Mathematics with Computer Science)

MCS 301 Semester III

Computer Networks

Paper – I

Unit I

Introduction – Definition and Uses of Computer Networks, PAN, LAN, MAN, WAN, Internet, Protocol Hierarchies, Layering Scenario, The OSI Reference Model, The TCP/IP Reference Model, Comparison of OSI and TCP/IP Reference Models, Network Standardization, Metric Units. Physical Layer – Overview of guided transmission media and wireless transmission. Data Link Layer – Design Issues, Error Detection and Correction, Elementary Data Link Layer Protocols, Sliding Window Protocol.
Multiple Access Protocols – ALOHA, CSMA, Collision-Free Protocols, Ethernet – Ethernet MAC Sublayer Protocol. Data Link Layer Switching – use of bridges, learning bridges, Spanning Tree bridges, Repeaters, Hubs, Bridges, Switches, Routers and Gateways.

Unit II

Network Layer – Network Layer Design Issues, store and forwards packet switching, connectionless service and connection-oriented service, Routing Algorithms – The Optimality Principle, Shortest Path, Flooding, Distance Vector Routing, Count-to-Infinity Problem, Hierarchical Routing, Congestion Control Algorithms.

Unit III

Overview of Quality of Service, Internetworking – Tunnelling, Internetwork Routing, Packet Fragmentation, IPv4, IPv6 Protocol, IP Addresses, CIDR, IMCP, ARP, RARP, DHCP. Transport Layer – Services Provided to the Upper Layers, Elements of Transport Protocol-Addressing, Connection Establishment, Connection Release, Error Control and Flow Control, Crash Recovery.

Unit IV

The Internet Transport Protocols: UDP – Introduction, RPC, Real-Time Transport Protocols, TCP – Introduction, The TCP Service Model, The TCP Segment Header, The Connection Establishment, The TCP Connection Release, The TCP Connection Management Modelling, TCP Sliding Window, The TCP Congestion Control, The Future of TCP. Application Layer – Introduction, Providing Services, Applications Layer Paradigms, Client server model, Standard Client-Server application-HTTP, FTP, Electronic Mail, TELNET, DNS, SSH.

Text books:

1. Computer Networks, Andrew S. Tanenbaum, David J Wetherall, Pearson Education, 5th Edition.
2. Computer Networks A Top-Down Approach, Behrouz A Forouzan, Firouz Mosharraf, TMH.

M.Sc. (Mathematics with Computer Science)

Computer Networks

MCS 351 Paper I Semester III

Practical Questions

1) Socket pair system call usage in IPC
2) Echo concurrent Stream Server
3) Echo concurrent stream client
4) Listener
5) Talker
6) Socket options using signals
7) TCP time service
8) UDP time service
9) Ping service
10) Route tracing program
11) Shortest path routing Algorithm implementation
12) I/O Multiplexing using select system call
13) Distance Vector Routing Implementation
14) ICMP Error Message simulations

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Mathematics with Computer Science)

MCS 302 Semester III

Mathematical Methods

Paper – II

Unit I

Existence and Uniqueness of solution of $\frac{dy}{dx} = f(x,y)$. The method of successive approximation- Picard's theorem- Sturm-Liouville's boundary value problem.
Partial Differential Equations: Origins of first-order PDES-Linear equation of first-order-Lagrange's method of solving PDE of $P_p+Qq = R$ – Non-Linear PDE of order one-Charpit method- Linear PDES with constant coefficients.

Unit II

Partial Differential Equations of order two with variable coefficients- Canonical form Classification of second order PDE- separation of variable method solving the one-dimensional Heat equation and Wave equation- Laplace equation.

Unit III

Power Series solution of O.D.E._– Ordinary and Singular points- Series solution about an ordinary point -Series solution about Singular point-Frobenius Method.
Lagendre Polynomials:_Lengendre's equation and its solution- Lengendre Polynomial and its properties- Generating function-Orthogonal properties- Recurrance relations- Laplace's definite integrals for P_n (x)- Rodrigue's formula.

Unit IV

Bessels Functions:_Bessel's equation and its solution- Bessel function of the first kind and its properties- Recurrence Relations- Generating function- Orthogonality properties.
Hermite Polynomials:_Hermite's equation and its solution- Hermite polynomial and its properties- Generating function- Alternative expressions (Rodrigue's formula)- Orthogonality properties- Recurrence Relations.

Text Books:

1. "Elements of Partial Differential Equations", By Ian Sneddon, Mc.Graw-Hill International Edition.
2. "Text book of Ordinary Differential Equation", By S.G.Deo, V. Lakshmi Kantham, V. Raghavendra, Tata Mc.Graw Hill Pub. Company Ltd.
3. "Ordinary and Partial Differential Equations", By M.D. Raisingania, S. Chand Company Ltd., New Delhi.

M.Sc. (Mathematics with Computer Science)

Mathematical Methods

MCS 352 Paper II Semester III

Practical Questions

1. Compute the first three successive approximations for the solution of the initialvalue problem $\frac{dx}{dt} = x^2, \quad x(0) = 1.$
2. Solve $yp = 2yx + \log q.$
3. Solve $yzp + zxq = xy$ with usual notations.
4. Explain Strum-Liouille's boundary value problems.
5. Classify the equation $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0.$
6. Solve $r + t + 2s = 0$ with the usual notations.
7. Find the particular integral of the equation $(D^2 - D)Z = e^{2x+y}$.
8. Solve in series the equation $xy'' + y' - y = 0$.
9. Solve $y'' - y = x$ using power series method.
10. Solve the Froenius method $x^2 y'' + 2x^2 y' - 2y = 0$.
11. Solve in series $2xy'' + 6y' + y = 0.$
12. Prove that $J_{-n}(x) = (-1)^n J_n(x)$ where n is an integer.
13. Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$.
14. Prove that $H_n(-x) = (-1)^n H_n(x).$
15. Show that $H_{2n+1}(0) = 0.$
16. Show that $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x).$
17. Solve $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$; *with* $u(x,0) = 4e^{-x}$ using separation of variable method.
18. Find the surface passing through the parabolas $Z = 0,\ y^2 = 4ax\ and\ Z = 1,\ y^2 = -4ax$ and satisfying the equation $xr + zp = 0$.
19. Find the surface satisfying $t = 6x^2 y$ containing two lines y = 0 = z and y = 2 = z .
20. Reduse the equation $x^2 r - y^2 t + px - qy = x^2$ in the canonical form.

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. (Mathematics with Computer Science)

MCS – 303A Semester III

Advanced RDBMS

Paper III (A)

Unit I

Introduction – Database System Applications– Purpose of database systems – View of Data – Database Languages – Relational Databases – Database Design, Database Architecture – Database Users and Administrators – Structure of Relational Databases – Database Schema – Keys – Schema Diagrams – Relational Query Languages – The Relational Algebra – The Tuple Relational Calculus – The Domain Relational Calculus.

Unit II

Overview of the SQL Query Language – SQL Data Definition – Basic Structure of SQL Queries, SQL Additional Basic Operations – Set Operations – Null Values – SQL Aggregate Functions – Nested Subqueries – Modification of the Database. The Entity-Relationship Model – Constraints – Removing Redundant Attributes in Entity Sets – Entity-Relationship Diagrams – Reduction to Relational Schemas – Entity-Relationship Design Issues – Extended E-R Features.

Unit III

Features of Good Relational Designs – Atomic Domains and First Normal Form – Decomposition Using Functional Dependencies – 2NF, 3NF, BCNF, 4NF, 5NF. SQL – views – Integrity constraints – Functions and Procedures – Triggers. File Organization – Organization of Records in Files – Data-Dictionary Storage – Indexing and Hashing – Basic Concepts – Ordered Indices – B^+-Tree Index Files – B^+-Tree Extensions – Static Hashing – Dynamic Hashing – Bitmaps Indices.

Unit IV

Transaction Concept – A Simple Transaction Model – Transaction Atomicity and Durability – Transaction Isolation – Serializabitlity – Transaction Isolation and Atomicity – Transaction Isolation Levels – Implementation of Isolation Levels. Concurrency Control –Lock based protocols – Deadlock Handling – Multiple Granularity – Time-stamp based protocols – Validation based protocol – Multiversion Schemes. Recovery System – Failure Classification – Storage, Recovery and Atomicity, Recovery Algorithm, Buffer Management.

Text Books:

1. Abraham Silberschatz, Henry F. Korth, S. Sudarshan, Database System Concepts
2. Raghu Ramakrishnan, Johannes Gehrke, Database Management Systems
3. Ramez Elmasri, Shamkant B. Navathe, Fundamentals of Database Systems
4. Jeffrey A. Hoffer, V. Ramesh, Heikki Topi, Modern Database Management
5. Thomas M. Connolly, Carolyn E. Begg, Database Systems–A Practical Approach to Design, Implementation, and Management

M.Sc. (Mathematics with Computer Science)

Advanced RDBMS

MCS 353(A) Paper III (A) Semester III

Practical Questions

I. Create the tables using the university schema and write the following queries in SQL

1. Find the titles of courses in the Comp. Sci. department that have 3 credits.
2. Find the names of all instructors in the Computer Science department who have salary greater than $70,000.
3. Retrieve the names of all instructors, along with their department names and department building name.
4. For all instructors in the university who have taught some course, find their names and the course ID of all courses they taught using where clause.
5. For all instructors in the university who have taught some course, find their names and the course ID of all courses they taught using natural join.
6. Find the names of all departments whose building name includes the substring 'Watson'."
7. Find the names of instructors with salary amounts between $90,000 and $100,000.
8. Same above query using between.
9. Find all courses taught in the Spring 2010 semester.
10. Find all courses taught either in Fall 2009 or in Spring 2010, or both.
11. Find the all courses taught in the Fall 2009 as well as in Spring 2010.
12. Find all courses taught in the Fall 2009 semester but not in the Spring 2010 semester.
13. Find the average salary of instructors in the Computer Science department.
14. Find the total number of instructors who teach a course in the Spring 2010 semester."
15. Find the number of tuples in the course relation.
16. Find the average salary in each department.
17. Find the number of instructors in each department who teach a course in the Spring 2010 semester
18. Find the average salary of instructors in those departments where the average salary is more than $42,000.
19. For each course section offered in 2009, find the average total credits (tot cred) of all students enrolled in the section, if the section had at least 2 students.
20. Find all the courses taught in the Fall 2009 semester but not in the Spring 2010 sem.
21. Find the total number of (distinct) students who have taken course sections taught by the instructor with *ID* 110011"
22. Find the names of all instructors that have a salary value greater than that of each instructor in the Biology department.
23. Find the departments that have the highest average salary
24. Find all students who have not taken a course using natural left outer join.
25. Find all students who have not taken a course using natural right outer join.
26. Create a view that lists all course sections offered by the Physics department in the Fall 2009 semester with the building and room number of each section.
27. Find all Physics courses offered in the Fall 2009 semester in the Watson building in above view.
28. Increase the salary of each instructor in the Comp. Sci. department by 10%.
29. Delete all courses that have never been offered (i.e., do not occur in section relation).
30. Insert every student whose tot_cred attribute is greater than 100 as an instructor in the same Department, with a salary of $10,000.

II. Given relation schemas are

Sailors(sid : integer, sname : string, rating : integer, age : real)
Boats(bid : integer, bname : string, color : string)
Reserves(sid : integer , bid : integer, day : date)

1. Find the names and ages of all sailors.
2. Find all sailors with a rating above 7.
3. Find the names of sailors who have reserved boat 103.
4. Find the sids of sailors who have reserved a red boat.
5. Find the names of sailors who have reserved a red boat.
6. Find the colors of boats reserved by Lubber.
7. Find the names of sailors who have reserved at least one boat.
8. Find the names of sailors who have reserved at least two boats.
9. Compute increments for the ratings of persons who have sailed two different boats on the same day.
10. Find the ages of sailors whose name begins and ends with B and has at least three characters.
11. Find the names of sailors who have reserved a red or a green boat.
12. Find the names of sailors who have reserved a red and a green boat.
13. Find the sids of all sailors who have reserved red boats but not green boats.
14. Find all sids of sailors who have a rating of 10 or have reserved boat 104.
15. Find the names of sailors who have not reserved a red boat.
16. Find sailors whose rating is better than some sailor called Horatio.
17. Find sailors whose rating is better than every sailor called Horatio.
18. Find the names of sailors who have reserved all boats.
19. Find the names of sailors who have reserved at least two boats.
20. Find the names of sailors who have reserved all boats called Interlake.
21. Find sailors who have reserved all red boats.
22. Find the sailor name, boat id, and reservation date for each reservation.
23. Find the sids of sailors with age over 20 who have not reserved a red boat.
24. Find the average age of all sailors.
25. Find the average age of sailors with a rating of 10.
26. Find the name and age of the oldest sailor.
27. Count the number of different sailor names.
28. Find the names of sailors who are older than the oldest sailor with a rating of 10.
29. Find the sailors with the highest rating.
30. Find the age of the youngest sailor for each rating level.
31. Find age of the youngest sailor who is eligible to vote for each rating level with at least 2 such sailors.
32. Find the average age of sailors for each rating level that has at least two sailors.
33. For each red boat, find the number of reservations for this boat.
34. Find the average age of sailors who are of voting age (i.e., at least 18 years old) for each rating level that has at least two sailors.
35. Delete the records of sailors who have rating 8 (deleting some rows in a table).

III. Write a program to explain views in SQL.

IV. Write a program to explain functions in SQL.

V. Write a program to explain procedures in SQL.

VI. Write a program to explain triggers in SQL.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Mathematics with Computer Science)

MCS 303B Semester III

Design and Analysis Algorithm

Paper – III (B)

Unit I

Introduction: definition and characteristics of algorithm, different notations for representing algorithms, writing an algorithms for simple problems, analysing algorithms-asymptotic notations, time and space complexity. Elementary Data Structures: stacks and queues, trees, heaps and heap sort, graph representations, hashing, sets representation-UNION, FIND operation.

Unit II

Divide–and–Conquer: general method, binary search, finding maximum and minimum, merge sort, quick sort, selection sort, strassen's matrix multiplication. The Greedy Method: general method, knapsack problem, optimal storage on tapes, job sequencing with deadlines, optimal merge pattern (huffman codes), minimum spanning trees-prime's, kruskal's, single source shortest pattern.

Unit III

Dynamic Programming: introduction, all-pairs shortest paths, optimal binary search trees, 0/1-knapsack, reliability design, travelling sales man problem. Traversal Techniques: binary tree traversals-preorder, inorder, postorde, breadth first search, depth first search, bi-connected components and articulation point.

Unit IV

Back Tracking Technique: introduction, 4-queens and 8-queens problems, graph colouring, Hamiltonian cycles, Knapsack problem. Branch-and-Bound Technique: introduction, 0/1 Knapsack problems, travelling sales person problem. NP–Hard and NP-Complete Problems: basic concepts, cook's theorem, NP-Hard graph problems and scheduling problems, decision problem, node covering theorem.

Text Book

1. E. Horowitz and S. Sahini, Fundamentals of Computer Algorithms, Galgotia Publications, 1984.
2. A.V. Aho, J.V. Hopcraft and J.D. Ullmann, The design and analysis of computer Algorithm, Addison Wesley Publications Company 1974

M.Sc. (Mathematics with Computer Science)
Design and Analysis algorithm

MCS 353B Paper III (B) Semester III

Practical Questions

1. Write programs to implement the Stack using arrays.
2. Write programs to implement the Queue using arrays.
3. Write programs to implement the Stack using liked lists.
4. Write a program to find the given number in a list using Binary Search.
5. Write a program for sorting the given list using Selection Sort.
6. Write a program for sorting the given list using Merge Sort.
7. Write a program for sorting the given list using Quick Sort.
8. Write a program for sorting the given list using Heap Sort.
9. Write a program to implement the Kruskal's Algorithm.
10. Write a program to implement the Dijkstra's Algorithm.
11. Write a program to the implementation BFS.
12. Write a program to the implementation DFS.
13. Write a program to solve travelling sales man problem.
14. Write a program to solve knapsack problem.
15. Write a program to find the Hamiltonian circuit for a weighted graph.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Maths with Computer Science)

MCS – 303C Semester III

Object Oriented Analysis and Design

Paper III (C)

Unit I

Introduction to Object-Oriented Analysis and Design, A Short Example., Overview of UML and Visual agile Modeling, History.
Overview of UP, Iterative and Evolutionary Development, Waterfall Lifecycle, Iterative and Evolutionary Analysis and Design, Risk-Driven and Client-Driven Iterative Planning, Agile Methods and Attitudes, Agile Modeling, Agile UP, UP Phases, UP Disciplines, the UP Development Case.
Case Study Strategy: Iterative Development + Iterative learning .Case one: The NextGen POS System, Case Two: The Monopoly Game System. Definition of Inception, Inception Artifacts Definition: Requirements, Evolutionary vs. Waterfall Requirements, Skillful Means to Find Requirements, Types and Categories of Requirements, UP Requirements Artifacts and its Organization. Definition of Actors, Scenarios, and Use Cases, Use Cases and the Use-Case Model. Motivation for the Use Cases, Three Kinds of Actors, Three Common Use Case Formats, Example: Process Sale, Fully Dressed Style. Guideline to find Use Cases. Applying UML: Use Case Diagrams, Applying UML: Activity Diagrams, Other Benefits of Use Cases, Example: Monopoly Game, Process: How to Work With Use Cases in Iterative Methods?
Showing the Supplementary Specification, Glossary, Vision & Business Rules, Evolutionary Requirements in Iterative Methods. Iteration 1 Requirements and Emphasis: Core OOA/D Skills. Process: Inception and Elaboration. Process: Planning the Next Iteration.

Unit II

Domain Model, Motivation for the Creation of a Domain Model, Guideline to Create a Domain Model and to find Conceptual Classes, Example: Find and Draw Conceptual Classes. Agile Modeling-Sketching a Class Diagram. Guideline: Agile Modeling-Maintain the Model in a Tool? Guideline: Report Objects-Include 'Receipt' in the Model? A Common Mistake with Attributes vs. Classes. Associations, Attributes in domain model. Iterative and Evolutionary Domain Modeling. Example: NextGen SSD. System Sequence Diagrams, Motivation, Applying UML: Sequence Diagrams. Relationship between SSDs and Use Cases, Naming the System Events and Operations, Modeling SSDs Involving Other External Systems, Example: Monopoly SSD. Iterative and Evolutionary SSDs.
System Operation, Contracts Usefulness, Guideline to Create and Write Contracts. Example: NextGen POS Contracts. Example: Monopoly Contracts. Applying UML: Operations, Contracts, and the OCL. Process: Operation Contracts within the UP. Iterative process. Provoking Early Change. Logical Architecture, Focus in the Case Studies, Software Architecture, Applying UML: Package Diagrams. The Model-View Separation Principle. Connection between SSDs, System Operations, and Layers, Example: NextGen Logical Architecture and Package Diagram. Example: Monopoly Logical Architecture. Agile Modeling and Lightweight UML Drawing. UML CASE Tools. Static and Dynamic Modeling, the Importance of Object Design Skill over UML Notation Skill. Other Object Design Techniques: CRC Cards. Sequence & Communication Diagrams. Common UML Interaction Diagram Notation. Basic Sequence Diagram Notation. Basic Communication Diagram Notation. Applying UML: Class Diagram Notation. Relationship between Interaction and Class Diagrams.

Unit III

UML versus Design Principles. GRASP: A Methodical Approach to Basic OO Design. Connection between Responsibilities, GRASP, and
UML Diagrams. Patterns, A Short Example of Object Design with GRASP. Applying GRASP to Object Design. Creator. Information Expert (or Expert). Low Coupling. Controller. High Cohesion. Use Case Realization, Use Case Realizations for the NextGen Iteration. Use Case Realizations for the Monopoly Iteration. Iterative and Evolutionary Object Design. Visibility between Objects, Visibility, Four kinds of visibility. Programming and Iterative, Evolutionary Development, Mapping Designs to Code , Creating Class Definitions from DCDs, Creating Methods from Interaction Diagrams, Collection Classes in Code, Exceptions and Error Handling, Defining the Sale. MakeLineItem Method. Order of Implementation. Test-Driven or Test-First Development. Introduction to the NextGen POS Program Solution. Test-Driven Development. Refactoring. Forward, Reverse, and Round-Trip Engineering. Common Report of Valuable Features, suggestions for choosing a UML tool, suggestions on how to integrate UML wall sketching and tools. Case Study: NextGen
POS. Case Study: Monopoly. From Iteration 1 to 2. Iteration-2 Requirements and Emphasis: Object Design and Patterns

Unit IV

Polymorphism. Pure Fabrication. Indirection. Protected Variations. Adapter (GoF). Factory. Singleton (GoF). Strategy (GoF). Composite (GoF) and Other Design Principles. Facade (GoF). Observer/Publish-Subscribe/Delegation Event Model (GoF).NextGen POS. Monopoly. UML Activity Diagram Notation. Guidelines. Example: NextGen Activity Diagram. Definitions: Events, States, and Transitions. UMLState Machine Diagram Notation. Example: NextGenUseCaseState Machine Diagram. The include Relationship. Terminology: Concrete, Abstract, Base, and Addition Use Cases. The extend Relationship. The generalize Relationship. Use Case Diagrams. New Concepts for the NextGen Domain Model. Generalization. Defining Conceptual Super classes and Subclasses. NextGen POS Conceptual Class Hierarchies. Abstract Conceptual Classes. Modeling Changing States. Class Hierarchies and Inheritance in Software. Association Classes. Aggregation and Composition. Association Role Names. Roles as Concepts versus Roles in Associations. Derived Elements. Qualified Associations. Reflexive Associations. Using Packages to Organize the Domain Model. Example: Monopoly Domain Model Refinements. Architectural Analysis, Variation and Evolution Points. Common Steps in Architectural Analysis. Identification and Analysis of Architectural Factors. Example: Partial NextGen POS Architectural Factor Table.

Text Book

T1 Larman , Craig, Applying UML and Patterns: An Introduction to Object-Oriented Analysis and Design and Iterative Development, Pearson Education, 3rd Ed., 2004.

Reference Book (S)

R1. Barclay Savage, Object Oriented Design with UML and JAVA, Elsevier, 2008.
R2. Brown, D.W., An Introduction to Object-Oriented Analysis, Wiley, 2nd Ed., 2004.
R3. Mark Priestley, Practical Object Oriented Design with UML, TMH, 2nd Ed., 2005.
R4. Michael Bleha, James Rambaugh, Object-Oriented Modelling & Design with UML, Pearson, 2nd Ed., 2005.
R5. Bahrami A., Object Oriented Systems Development using Unified Modeling Language, McGraw Hill, 1999.
R6. Grady Booch et al., Unified Modeling Language User Guide, Pearson Education, 1999
R7. Martin Fowler et al., UML Distilled, Pearson Education, 2000
R8. Rebecca Wirfs-Brock et al., Designing Object-Oriented Software, PHI, 1996
R9. Bruegge B., Object-Oriented Software Engineering, Pearson, 2000.

M.Sc. (Mathematics with Computer Science)

Object Oriented Analysis and Design

MCS 353C | Paper III (C) | Semester III

1. Describe all notation of use case diagram.
2. Draw use case diagram for following systems
   - Library management system
   - Hospital management system
   - Online bus/railway reservation system
   - Hotel management system
3. Describe all notation of class diagram.
4. Draw class diagram for following systems
   - Library management system
   - Hospital management system
   - Online bus/railway reservation system
   - Hotel management system
5. Describe all notation of state diagram.
6. Draw state diagram for following systems
   - Library management system
   - Hospital management system
   - Online bus/railway reservation system
   - Hotel management system
7. Describe all notation of sequence diagram.
8. Draw sequence diagram for following systems
   - Library management system
   - Hospital management system
   - Online bus/railway reservation system
   - Hotel management system
9. Describe all notation of activity diagram.
10. Draw activity diagram for following systems
    - Library management system
    - Hospital management system
    - Online bus/railway reservation system
    - Hotel management system

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY

M.Sc. (Mathematics with Computer Science)

MCS – 303D Semester III

Robotics and Artificial Intelligence

Paper III (D)

Unit I

Introduction to Robotics: What is a Robot? Definition, History of Robots: Control Theory, Cybernetics, Grey Walter Tortoise, Analog Electronic Circuit, Reactive Theory, Braitenberg's Vehicle, Artificial Intelligence, Vision Based Navigation, Types of Robot Control. Robot Components: Embodiment, Sensors, States, Action, Brains and Brawn, Autonomy, Arms, Legs, Wheels, Tracks, and What really drives them effectors and actuators: Effector, Actuator, Passive and Active Actuation, Types of Actuator, Motors, Degree of freedom Locomotion: Stability, Moving and Gaits, Wheels and Steering, Staying on the path. Manipulators: End effectors, Teleoperation, Why is manipulation hard? Sensors: Types of Sensors, Levels of Processing, Passive and Active sensors, Switches, Light sensors, Resistive position sensor.

Unit II

Sonar, Lasers and Cameras: Ultrasonic and Sonar sensing, Specular Reflection, Laser Sensing, Visual Sensing, Cameras, Edge Detection, Motion Vision, Stereo Vision, Biological Vision, Vision for Robots, Feedback or Closed Loop Control: Example of Feedback Control Robot, Types of feedback control, Feed forward or Open loop control.

Unit III

Languages for Programming Robot: Algorithm, Architecture, The many ways to make a map, What is planning, Cost of planning, Reactive systems, Action selection, Subsumption architecture, How to sequence behavior through world, hybrid control, Behavior based control and Behavior Coordination, Behavior Arbitration, Distributed mapping, Navigation and Path planning.

Unit IV

Artificial Intelligence: Introduction, State space search: Generate and test, Simple search, Depth First Search (DFS), Breadth First Search (DFS), Comparison and quality of solutions. Heuristic Search: Heuristic functions, Best First Search (BFS), Hill Climbing, Local Maxima, Beam search, Tabu search. Finding Optimum paths: Brute force, branch & bound, refine search, Dijkstra's algorithm, A* algorithm. Admissibility of A* algorithm.

Text book:

1. The Robotics Primer by Maja J Matarić, MIT press Cambridge,
2. A First course in Artificial Intelligence, Deepak Khemani, Tata McGraw Hill Education

References:

1. Artificial Intelligence: A Modern Approach, 3e, Stuart Jonathan Russell,
2. Artificial Intelligence Illuminated, Ben Coppin, Jones and Bartlett Publishers Inc
3. Artificial Intelligence A Systems Approach, M Tim Jones, Firewall media
4. Artificial Intelligence-Structures and Strategies for Complex Problem Solving, 4/e, George Lugar, Pearson Education

M.Sc. (Mathematics with Computer Science)

Robotics and Artificial Intelligence

MCS 353(D) Paper III (D) Semester III

Practical Questions

1. Write a program to create a robot
   i. With gear
   ii. Without gear and move it forward, left, right
2. Write a program to create a robot with a two motor and move it forward, left, right
3. Write a program to do a square using a while loop, doing steps with a for loop, to change directions based on condition, controlling motor speed using switch case,
4. Write a program to create a robot with light sensors to follow a line
5. Write a program to create a robot that does a circle using 2 motors
6. Write a program to create a path following robot
7. Write a program to register obstacles
8. Write a program to implement Breadth First Search (BFS) algorithm for a iven standard problem
9. Write a program to implement Hill Climbing algorithm for a given standard problem.
10. Write a program to implement A* search algorithm for a given standard problem.

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. (Mathematics with computer science)

MCS 304A Semester III

Elementary Number Theory

Paper – IV (A)

Unit I

The Division Algorithm – Number Patterns- Prime and Composite Numbers- Fibonacci and Lucas' numbers- Fermat Numbers- GCD-The Euclidean Algorithm- The Fundamental Theorem of Arithmetic- LCM- Linear Diophantine Equations.

Unit II

Congruences- Linear Congruences- The Pollard Rho Factoring Method- Divisibility Tests- Modular Designs- Check Digits- The Chinese Remainder Theorem- General Linear Systems- 2X2 Systems.

Unit III

Wilson's Theorem- Fermat's Little Theorem- Pseudo primes- Euler's Theorem- Euler's Phi function Revisisted- The Tau and Sigma Functions- Perfect Numbers- Mersenne Primes- The Mobius Function.

Unit IV

The Order of a Positive Integer- Primality Tests- Primitive Roots for Primes- Composites with Primitive roots- The Algebra of Indices- Quadratic Residues- the Legendre Symbol- Quadratic Reciprocity- the Jacobi Symbol.

Text Books:

1. Thomas Koshy, Elementary Number Theory with Applications.

M.Sc. (Mathematics with Computer Science)

Elementary Number Theory

MCS 354A Paper IV (A) Semester III

Practical Questions

1

Find the positive integer a if $[a, a+1] = 132$.

2

Find the twin primes p and q such that $[p, q] = 323$.

3

The LDE $ax + by = c$ is solvable if and only if $d|c$, where $d = (a, b)$. If x_0, y_0 is a particular solution of the LDE, then all its solutions are given by

$$x = x_0 + \left(\frac{b}{d}\right)t \quad \text{and} \quad y = y_0 - \left(\frac{a}{d}\right)t$$

where t is an arbitrary integer.

4

Solve the LDE $1076x + 2076y = 3076$ by Euler's method.

5

Find the general solution of each LDE

$2x + 3y = 4$

$12x + 13y = 14$

6

Determine the number of incongruent solutions of each linear congruence.

$12x \equiv 18 \pmod{15}$

$28u \equiv 119 \pmod{91}$

$49x \equiv 94 \pmod{36}$

7

Using congruences, solve each LDE.

$3x + 4y = 5$

$15x + 21y = 39$

8

Using the Pollard rho method, factor the integer 3893.

9

Prove that the digital root of the product of twin primes, other than 3 and 5, is 8.

10

Using the CRT, solve Sun-Tsu's puzzle:

$$x \equiv 1 \pmod 3, \quad x \equiv 2 \pmod 5, \quad \text{and} \quad x \equiv 3 \pmod 7$$

11

Prove each, where p is a prime.

Let p be odd. Then $2(p-3)! \equiv -1 \pmod{p}$.

$(p-1)(p-2)\cdots(p-k) \equiv (-1)^k k! \pmod{p}$, where $1 \le k < p$.

12

Find the remainder when 24^{1947} is divided by 17.

13

Let p be any odd prime and a any nonnegative integer. Prove the following.

$1^{p-1} + 2^{p-1} + \cdots + (p-1)^{p-1} \equiv -1 \pmod{p}$

$1^p + 2^p + \cdots + (p-1)^p \equiv 0 \pmod{p}$

14

Verify each.

$(12+15)^{17} \equiv 12^{17} + 15^{17} \pmod{17}$

$(16+21)^{23} \equiv 16^{23} + 21^{23} \pmod{23}$

15

Find the remainder when 245^{1040} is divided by 18.

16

Evaluate $(-4/41)$ and $(-9/83)$.

17

Verify that $9973 | (2^{4986} + 1)$.

18

Prove that there are infinitely many primes of the form $4n + 1$.

19

Show that $1! + 2! + 3! + \cdots + n!$ is never a square, where $n > 3$.

20

Prove that there are infinitely many primes of the form $10k - 1$.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Mathematics with computer science)

MCS 304B   Semester III

Integral Equations and Calculus of Variations

Paper – IV (B)

Unit I

Volterra Integral Equations: Basic concepts - Relationship between Linear differential equations and Volterra Integral equations - Resolvent Kernel of Volterra Integral equation. Differentiation of some resolvent kernels - Solution of Integral equation by Resolvent Kernel - The method of successive approximations - Convolution type equations - Solution of Integro-differential equations with the aid of the Laplace Transformation – Volterra integral equation of the first kind - Euler integrals - Abel's problem - Abel's integral equation and its generalizations.

Unit II

Fredholm Integral Equations:Fredholm integral equations of the second kind – Fundamentals – The Method of Fredholm Determinants - Iterated Kernels constructing the Resolvent Kernel with the aid of Iterated Kernels - Integral equations with Degenerated Kernels. Hammerstein type equation - Characteristic numbers and Eigen functions and its properties. Green's function: Construction of Green's function for ordinary differential equations - Special case of Green's function - Using Green's function in the solution of boundary value problem.

Unit III

The Method of Variations in Problems with fixed Boundaries: Definitions of Functionals – Variation and Its properties - Euler's' equation – Fundamental Lemma of Calculus of Variation- The problem of minimum surface of revolution – Minimum Energy Problem Brachistochrone Problem - Variational problems involving Several functions - Functional dependent on higher order derivatives - Euler Poisson equation.

Unit IV

Functional dependent on the Functions of several independent variables - Euler's equations in two dependent variables - Variational problems in parametric form - Application of Calculus of Variation - Hamilton's principle - Lagrange's Equation, Hamilton's equations.

Text Books:

1. M. Krasnov, A. Kiselev, G. Makarenko, Problems and Exercises in Integral Equations
2. S. Swarup, Integral Equations
3. L.Elsgolts, Differential Equation and Calculus of Variations

M.Sc. (Mathematics with Computer Science)

Integral Equations and Calculus of Variations

MCS 354B Paper IV (B) Semester III

Practical Questions

1. From an Integral equation corresponding to the differential equation $y''' + xy'' + (x^2 - x)y = xe^x + 1$; $y(0) = y'(0) = 1$; $y''(0) = 1$
2. Convert the differential equation $y''' + xy'' + (x^2 - x)y = xe^x + 1$; with initial conditions $y(0) = y'(0) = 1$, $y''(0) = 0$; into Volterra's Integral Equations.
3. Solve the Integral Equations $\varphi''(x) + \varphi(x) + \int Sinh(x-t)\varphi(t)dt + \int_0^x Cosh(x-t)\varphi'(t)dt = Coshx$; $\varphi(0) = \varphi'(0) = 0$.
4. Solve the Integral Equations $\int_0^x \frac{\varphi(t)dt}{(x-t)^\alpha} = x^n$; $0 < \alpha < 1$;
5. With the aid of Resolvent Kernel, find the solution of the Integral equation $\varphi(x) = e^x \sin x + \int_0^x \frac{2+\cos x}{2+\cos t}\varphi(t)dt$
6. Solve the Integral Equations $\phi(x) - \lambda \int_0^1 \arccos t . \phi(t)dt = \frac{1}{\sqrt{1-x^2}}$
7. Find the Characteristic numbers and Eigen function of the Integral Equations $\phi(x) - \lambda \int_0^1 (45x^2 \log t - 9t^2 \log x)\phi(t)dt = 0$
8. **Applications of Green's function :** Construct Green's function for the homogeneous boundary value problem $y^{iv}(x) = 0$; $y(0) = y'(0) = 0$; $y(1) = y'(1) = 0$.
9. **Applications of Green's function :** Solve the Boundary Value problem $y^{iv}(x) = 1$; $y(0) = y'(0) = y''(1) = y'''(0) = 0$.
10. **Applications of Green's function :** Solve the Boundary Value problem $y'' + y = x^2$; $y(0) = y(\pi/2) = 0$.
11. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{1+y'^2}}{y}dx$
12. Test for an extremum the functional $v[y(x)] = \int_0^1 (xy + y^2 - 2y^2 y')dx$; $y(0) = 1$; $y(1)) = 2$.

13. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} (16y^2 - y''^2 + x^2)dx$

14. Determine the extremals of the functional $v[y(x)] = \int_{-l}^{l} (\frac{\mu}{2} y''^2 + \rho y)dx$ that satisfies the boundary conditions $y(-l) = y'(-l) = y(l) = y'(l) = 0$

15. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} (2yz - 2y^2 + y'^2 - z'^2)dx$

16. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} \left[y^2 + (y')^2 + \frac{2y}{Coshx} \right] dx$

17. Write the ***Ostrgradsky*** equation for the functional $v[z(x,y)] = \iint_D \left[\left(\frac{\partial z}{\partial x} \right)^2 - \left(\frac{\partial z}{\partial y} \right)^2 \right] dxdy$

18. **Applications of Hamilton's and Lagrange's equations:** Derive the equation of a vibrations of a Rectilinear Bar.

19. **Applications of Hamilton's and Lagrange's equations:** A particle of mass **m** is moving vertically under the action of gravity and a resistance force numerically equal to **k** times the displacement **x** from an equilibrium position. Obtain the Hamilton's and Euler's equation.

20. Use Hamilton's principle to find the equations for the small vibrations of a flexible stretching string of length ***l*** and tension **T** fixed at end points.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Mathematics with Computer Science)

MCS – 304 C Semester III

Operations Research

Paper IV(C)

Unit I

Formulation of Linear Programming problems, Graphical solution of Linear Programming problem, General formulation of Linear Programming problems, Standard and Matrix forms of Linear Programming problems, Simplex Method, Two-phase method, Big-M method, Method to resolve degeneracy in Linear Programming problem, Alternative optimal solutions. Solution of simultaneous equations by simplex Method, Inverse of a Matrix by simplex Method, Concept of Duality in Linear Programming, Comparison of solutions of the Dual and its primal.

Unit II

Mathematical formulation of Assignment problem, Reduction theorem, Hungarian Assignment Method, Travelling salesman problem, Formulation of Travelling Salesman problem as an Assignment problem, Solution procedure.
Mathematical formulation of Transportation problem, Tabular representation, Methods to find initial basic feasible solution, North West corner rule, Lowest cost entry method, Vogel's approximation methods, Optimality test, Method of finding optimal solution, Degeneracy in transportation problem, Method to resolve degeneracy, Unbalanced transportation problem.

Unit III

Concept of Dynamic programming, Bellman's principle of optimality, characteristics of Dynamic programming problem, Backward and Forward recursive approach, Minimum path problem, Single Additive constraint and Multiplicatively separable return, Single Additive constraint and Additively separable return, Single Multiplicatively constraint and Additively separable return.

Unit-IV

Historical development of CPM/PERT Techniques - Basic steps - Network diagram representation - Rules for drawing networks - Forward pass and Backward pass computations - Determination of floats - Determination of critical path - Project evaluation and review techniques updating.

Text Books:

[1] S. D. Sharma, Operations Research.
[2] Kanti Swarup, P. K. Gupta and Manmohan, Operations Research.
[3] H. A. Taha, Operations Research – An Introduction.

M.Sc. (Mathematics with Computer Science)

Operation Research

MCS 354(C) Paper IV(C) Semester III

Practical Questions

1. Find a geometrical interpretation and solution as well for the following LPP
Maximize $z=3x_1+ 5x_2$ subject to restrictions: $x_1+ 2x_2\leq 2000$, $x_1+ x_2\leq 1500$, $x_2\leq 600$ and $x_1\geq 0$, $x_2\geq 0$.
2. Solve the following LPP geometrically
Max. $z=8000 x_1+7000x_2$, subject to $3x_1+ x_2\leq 66$, $x_1+ x_2\leq 45$, $x_1\leq 20$, $x_2\leq 40$ and $x_1\geq 0$, $x_2\geq 0$.
3. Using Simplex method, solve the following LPP
Min. $z= x_1-3x_2+2x_3$ subject to $3x_1-x_2+3x_3\leq 7$, $-2x_1+4x_2\leq 12$, $-4x_1+3x_2+8x_3\leq 10$, and $x_1, x_2, x_3\geq 0$.
4. Use two-phase simplex method to solve the problem
Min. $z= x_1-2x_2-3x_3$ subject to the constraints $-2x_1+x_2+3x_3=2$, $2x_1+3x_2+4x_3=1$, and $x_1, x_2, x_3\geq 0$.
5. Solve by using Big-M Method for the problem
Max. $z=-2x_1-x_2$, subject to $3x_1+x_2=3$, $4x_1+3x_2\geq 6$, $x_1+2x_2\leq 4$ and $x_1\geq 0$, $x_2\geq 0$.
6. A department head has four subordinates, and four tasks to be performed. Subordinates differ in efficiency and tasks in their intrinsic difficulty. Time each man would take to perform each task is given in effectiveness matrix. How the tasks should be allocated to each person so as to minimize the total man-hour.

		Subordinates			
		I	II	III	IV
Tasks	A	8	26	17	11
	B	13	28	4	26
	C	38	19	18	15
	D	19	26	24	10

7. There are 5 jobs to be assigned on 5 machines and associated cost matrix is as follows:

		Machines				
		I	II	III	IV	V
Jobs	A	11	17	8	16	20
	B	9	7	12	6	15
	C	13	16	15	12	16
	D	21	24	17	28	26
	E	14	10	12	11	15

Find the optimum assignment and associated cost using assignment technique.

8. Given the matrix of set-up costs, show how to sequence the production so as to minimize the set up cost per cycle.

	A_1	A_2	A_3	A_4	A_5
A_1	∞	2	5	7	1
A_2	6	∞	3	8	2
A_3	8	7	∞	4	7
A_4	12	4	6	∞	5
A_5	1	3	2	8	∞

9. Consider the following transport problem

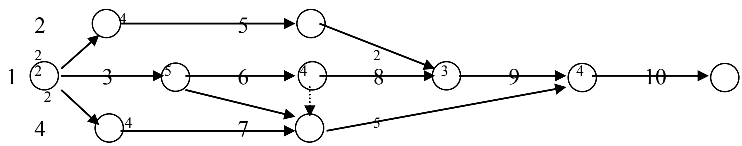
Source	Destination				Total
	D_1	D_2	D_3	D_4	
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
Total	20	40	30	10	100

Determine the initial feasible solution.

10. Find the initial basic feasible solution of the following transport problem by North West Corner Method.

		Warehouse				Factory Capacity
		W_1	W_2	W_3	W_4	
Factory	F_1	19	30	50	10	7
	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
Warehouse Requirement		5	8	7	14	34

11. Find the value of max $(y_1y_2y_3)$ subject to $y_1+y_2+y_3\geq 5$ and $y_1, y_2, y_3\geq 0$.
12. Minimize $z=y_1^2+ y_2^2+ y_3^2$ subject to $y_1+y_2+y_3\geq 15$ and $y_1, y_2, y_3\geq 0$.
13. Use dynamic programming to show that $-\sum_{i=1}^{n} p_i \log p_i$, subject to $\sum_{i=1}^{n} p_i = 1$ is maximum, when $p_1=p_2=\dots=p_n=1/n$.
14. Use the principle of optimality to find the maximum value of $z=b_1x_1+b_2x_2+\dots+b_nx_n$ where $x_1+ x_2+\dots+ x_n=c$, and $x_1, x_2,\dots, x_n\geq 0$, $b_1, b_2,\dots, b_n>0$.
15. Solve the following problem using dynamic programming: Minimize $z= y_1^2+ y_2^2+\dots+y_n^2$ subject to the constraints $y_1= y_2=\dots= y_n=b$ and $y_1, y_2,\dots, y_n\geq 0$.
16. Consider a project given by the following Network diagram. Numbers along various activities represent the normal time of completion of that activity D_{ij}



Find minimum time of completion of the project.

17. A project has the following time schedule (in months)

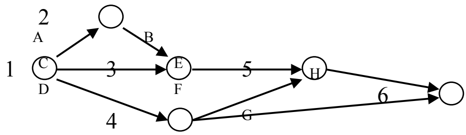
Activity	1→2	1→3	1→4	2→5	3→6	3→7	4→6	5→8	6→9	7→8	8→9
Normal Duration	2	2	1	4	8	5	3	1	5	4	3

Draw the network diagram to represent the above project. Find the critical path also.

18. A project has the following time schedule. Draw the Network diagram to represent this project. Find the critical path also.

Activity	1→2	1→3	1→4	2→5	3→6	3→7	4→7	5→8	6→8	7→9	8→9	9→10
Normal Duration	2	2	2	4	5	8	4	2	4	5	3	4

19. A project is represented by following Network diagram.



The time estimate of activities are as given below.

Activity	A	B	C	D	E	F	G	H
t_0	4	5	8	2	4	6	8	3
t_p	8	10	12	7	10	15	16	7
t_m	5	7	11	3	7	9	12	5

Determine the minimum expected time of completion of the project. Also determine the critical path.

20. A project has the following time schedule

Activity	Time in weeks	Activity	Time in weeks
1 →2	4	5 →7	8
1 →3	1	6 →8	1
2 →4	1	7 →8	2
3 →4	1	8 →9	1
3 →5	6	8 →10	8
4 →9	5	9 →10	7
5 →6	4		

Construct a PERT network and compute

(i) T_E and T_L for each event

(ii) Float for each activity and

(iii) critical path and its duration.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
(Choice Based Credit System)
(w.e.f. The Academic year 2017–2018)

M.Sc. (Mathematics with Computer Science)

SEMESTER – IV

<table>
<tr><th>S. No.</th><th>Sub. Code.</th><th>Subject</th><th>Hrs./ Week</th><th>Internal Assessment</th><th>Semester Exam</th><th>Total Marks</th><th>Credits</th></tr>
<tr><td>1. Core</td><td>MCS401</td><td>Automata Theory</td><td>4</td><td>20</td><td>80</td><td>100</td><td>4</td></tr>
<tr><td>2. Core</td><td>MCS402</td><td>Discrete Mathematics</td><td>4</td><td>20</td><td>80</td><td>100</td><td>4</td></tr>
<tr><td rowspan="3">3. Elective</td><td>MCS403A</td><td>Computer Organization</td><td rowspan="3">4</td><td rowspan="3">20</td><td rowspan="3">80</td><td rowspan="3">100</td><td rowspan="3">4</td></tr>
<tr><td>MCS403B</td><td>Computer Graphics</td></tr>
<tr><td>MCS403C</td><td>Network Security</td></tr>
<tr><td>4. Project</td><td>MCS304</td><td>Project Work</td><td>6</td><td>--</td><td>--</td><td>150</td><td>6</td></tr>
<tr><td>5. Practical</td><td>MCS351</td><td>Automata Theory</td><td>4</td><td>--</td><td>50</td><td>50</td><td>2</td></tr>
<tr><td>6. Practical</td><td>MCS352</td><td>Discrete Mathematics</td><td>4</td><td>--</td><td>50</td><td>50</td><td>2</td></tr>
<tr><td rowspan="3">7. Practical</td><td>MCS353A</td><td>Computer Organization</td><td rowspan="3">4</td><td rowspan="3">--</td><td rowspan="3">50</td><td rowspan="3">50</td><td rowspan="3">2</td></tr>
<tr><td>MCS353B</td><td>Computer Graphics</td></tr>
<tr><td>MCS353C</td><td>Network Security</td></tr>
<tr><td colspan="3" align="right">Total</td><td>30</td><td></td><td></td><td>600</td><td>24</td></tr>
<tr><td>8. Seminar</td><td>MCS354</td><td>Seminar</td><td>2</td><td>--</td><td>--</td><td>25</td><td>1</td></tr>
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DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Mathematics with Computer Science)

MCS 401 Semester IV

Automata Theory

Paper – I

Unit I

Fundamentals – alphabets, strings, languages, problems, graphs, trees, Finite State Systems, definitions, Finite Automaton model, acceptance of strings, and languages, Deterministic finite automaton and Nondeterministic finite automaton, transition diagrams, transition tables, proliferation trees and language recognizers, equivalence of DFA's and NFA's.
Finite Automata with ε-moves, significance, acceptance of languages, ε-closure, Equivalence of NFA's with and without ε-moves, Minimization of finite automata, Two-way finite automata, Finite Automata with output–Moore and Melay machines.

Unit II

Regular Languages: regular sets, regular expressions, identity rules, constructing finite automata for a given regular expressions, conversion of finite automata to regular expressions. Pumping lemma of regular sets and its applications, closure properties of regular sets. Grammar Formalism: Regular grammars–right linear and left linear grammars, equivalence between regular linear grammar and finite automata, inter conversion, Context free grammar, derivation trees, sentential forms, right most and leftmost derivation of strings, ambiguity.

Unit III

Context Free Grammars: Simplification of Context Free Grammars, Chomsky normal form, Greiback normal form, Pumping lemma for context free languages and its applications, closure of properties of CFL (proofs omitted). Push Down Automata: PDA definition, model, acceptance of CFL, acceptance by final state and acceptance by empty state and its equivalence. Equivalence of PDA's and CFL's, inter-conversion. (Proofs not required).

Unit IV

Membership Algorithm (CYK Algorithm) for Context Free Grammars. Turing Machine: TM definition, model, design of TM, computable functions, unrestricted grammars, recursively enumerable languages. Church's hypothesis, counter machine, types of TM (proofs omitted). Linear bounded automata and Context sensitive language. Chomsky hierarchy of languages. Introduction to language processors, phases of a compiler, parsing.

Text Books:

1. J. E. Hopcroft, J. D. Ullman, Introduction to Automata Theory, Languages, and Computation
2. John C. Martin, Introduction to Languages and the Theory of Computation
3. Mishra, Chandrashekaran, Theory of Computer Science
4. A. V. Aho, Monica S. Lam, Ravi Sethi, J. D. Ullman, Compilers Principles, Techniques, & Tools

M.Sc. (Mathematics with Computer Science)

Automata Theory

MCS 451 | Paper I | Semester IV

Practical Questions

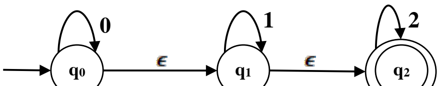
1. Construct the DFA for simple problems.
2. Convert the following NFA into its equivalent DFA.

States/Input	a	b
$\rightarrow q_0$	$\{q_1\}$	$\emptyset$
$* \ q_1$	$\{q_0, q_2\}$	$\{q_0, q_2\}$
q_2	$\{q_1\}$	$\{q_2\}$

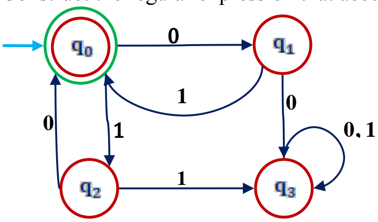
	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$* \ q_1$	$\emptyset$	$\{q_0, q_1\}$

	a	b
$\rightarrow * \ q_0$	$\{q_0, q_1\}$	$\{q_2\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\{q_2\}$	$\{q_1, q_2\}$

3. For the NFA with $\in$-moves M whose transition diagram is given below. Find an equivalent NFA without $\in$-moves M1.



4. Construct NFA with $\in$-moves for the RE: 01*+1 and 0*1+0
5. Construct DFA for the regular expressions: (a+b)*(aa+bb)(a+b)* and 10+(0+11)0*1
6. Construct the regular expression that accepts the FA.



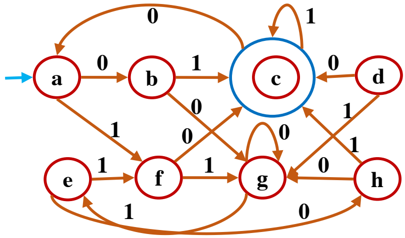
7. Find Mealy machine equivalent to following Moore machine.

	$\rightarrow q_0$	q_1	q_2	q_3
0	q_3	q_1	q_2	q_3
1	q_1	q_2	q_3	q_0
Output	0	1	0	0

8. Find Moore Machine equivalent to following Mealy Machine.

	0	Output	1	Output
$\rightarrow q_1$	q_1	1	q_2	0
q_2	q_4	1	q_4	1
q_3	q_2	1	q_3	1
q_4	q_3	0	q_1	1

9. Find the minimum number of states for the following DFA M.



10. Show that the following languages are not regular.
    $L = \{a^{n^2} / n \geq 1\}$; $L = \{a^p$ / p is prime$\}$; $L = \{ 0^i 1^i \mid i \geq 1 \}$; $L = \{ ww \mid w \in \{a, b\}^* \}$
11. State and prove the closure properties of regular sets.
12. Eliminate the null, unit and useless productions if any from the grammar G with productions: $A_1 \rightarrow A_2A_2A_3A_4$; $A_2 \rightarrow aA_2b|\varepsilon$; $A_3 \rightarrow aA_3|a$; $A_4 \rightarrow aA_4a|bA_4b|\varepsilon$
13. For the grammar G=({S, A, B}, {a, b}, P, S) that has productions $S \rightarrow bA|aB$, $A \rightarrow bAA|aS|a, B \rightarrow aBB|bS|b$, find an equivalent of compare in CNF.
14. Find the grammar in GNF equivalent to the CFG: $S \rightarrow AA|0; A \rightarrow SS|1$.
15. Find an equivalent GNF to the grammar to the CFG: $A \rightarrow BC$; $B \rightarrow CA|b$; $C \rightarrow AB|a$.
16. Determine whether the string $w = aabbb$ is in the language generated by the grammar $S \rightarrow AB, A \rightarrow BB|a, B \rightarrow AB|b$. Using CYK algorithm.
17. Find whether the string baaba is generated by or belongs to the grammar or not. The grammar is $S \rightarrow AB\ |BC$; $A \rightarrow BA\ |a$; $B \rightarrow CC \mid b$; $C \rightarrow AB\ |a$.
18. State the procedure for PDA construction from the grammar, construct a PDA equivalent to the following grammar: $S \rightarrow aA$, $A \rightarrow aABC|bB|a, B \rightarrow b, C \rightarrow c$.
19. Construct a pushdown automata equivalent to the following context free grammar: $S \rightarrow 0BB; B \rightarrow 0S|1S|0$.
20. Construct a CFG which accepts PDA $M=(\{q_0,q_1\},\{a,b\},\{z_0,z\},\delta,\ q_0,z_0,\phi)$ with transitions $\delta(q_0,b,z_0) = (q_0,zz_0)$, $\delta(q_0,\varepsilon,z_0) = (q_0,\varepsilon)$, $\delta(q_0,b,z) = (q_0,zz), \delta(q_0,a,z) = (q_1,z)$, $\delta(q_1,b,z) = (q_1,\varepsilon), \delta(q_1,a,z_0) = (q_0,z_0)$

    .
21. Prove that CFL is closed under union, concatenation and closure.
22. Design a Turing machine to accept the language L, where $L = \{0^n 1^n \mid n \geq 1\}$
23. Design a Turing machine to accept the language L, where $L = \{0^n 1^n 2^n \mid n \geq 1\}$
24. Design a Turing machine that compute x.y where x and y are given two positive integers.
25. Design a Turing machine that compute x+y where x and y are given two positive integers.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Mathematics with Computer Science)

MCS 402 Semester IV

Discrete Mathematics
Paper – II

Unit I

Mathematical Logic: propositional logic, propositional equivalences, predicates & quantifiers, rule of inference, direct proofs, proof by contraposition, proof by contradiction. Boolean Algebra: Boolean functions and its representation, logic gates, minimizations of circuits by using Boolean identities and K-map.

Unit II

Basic Structures: Sets representations, set operations, functions, sequences and summations. Division algorithm, modular arithmetic, solving congruences, applications of congruences. Recursion: Proofs by mathematical induction, recursive definitions, structural induction, generalized induction, recursive algorithms.

Unit III

Counting: Basic counting principle, inclusion-exclusion for two-sets, pigeonhole principle, permutations and combinations, Binomial coefficient and identities, generalized permutations and combinations. Recurrence Relations: introduction, solving linear recurrence relations, generating functions, principle of inclusion-exclusion, applications of inclusion-exclusion. Relations: relations and their properties, representing relations, closures of relations, equivalence relations, partial orderings.

Unit IV

Graphs: Graphs definitions, graph terminology, types of graphs, representing graphs, graph isomorphism, connectivity of graphs, Euler and Hamilton paths and circuits, Dijkstra's algorithm to find shortest path, planar graphs–Euler's formula and its applications, graph coloring and its applications. Trees: Trees definitions–properties of trees, applications of trees–BST, Haffman Coding, tree traversals: pre-order, in-order, post-order, prefix, infix, postfix notations, spanning tress–DFS, BFS, Prim's, Kruskal's algorithms.

Text Books:

1. Kenneth H. Rosen, Discrete Mathematics and its Applications
2. Ralph P. Grimaldi, Discrete and Combinatorial Mathematics
3. Stein, Drysdale, Bogart, Discrete Mathematics for Computer Scientists
4. J.P. Tremblay, R. Manohar, Discrete Mathematical Structures with Applications to Computer Science
5. Joe L. Mott, Abraham Kandel, Theoder P. Baker, Discrete Mathematics for Computer Scientists and Mathematicians

M.Sc. (Mathematics with Computer Science)

Discrete Mathematics

MCS 452 | Paper II | Semester IV

Practical Questions

1. Use K-maps to minimize these sum-of-products expansions.

   (a) $xy\overline{z} + x\overline{y}\,\overline{z} + \overline{x}yz + \overline{x}\,\overline{y}\,\overline{z}$
   (b) $x\overline{y}z + x\overline{y}\,\overline{z} + \overline{x}yz + \overline{x}\,\overline{y}z + \overline{x}\,\overline{y}\,\overline{z}$
   (c) $xyz + xy\overline{z} + x\overline{y}z + x\overline{y}\,\overline{z} + \overline{x}yz + \overline{x}\,\overline{y}z + \overline{x}\,\overline{y}\,\overline{z}$
   (d) $xy\overline{z} + x\overline{y}\,\overline{z} + \overline{x}\,\overline{y}z + \overline{x}\,\overline{y}\,\overline{z}$

2. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.
   a) Something is not in the correct place.
   b) All tools are in the correct place and are in excellent condition.
   c) Everything is in the correct place and in excellent condition.
   d) Nothing is in the correct place and is in excellent condition.
   e) One of your tools is not in the correct place, but it is in excellent condition.
3. Show that the premises "Everyone in this discrete mathematics class has taken a course in computer science" and "Marla is a student in this class" imply the conclusion "Marla has taken a course in computer science."
4. Show that the premises "A student in this class has not read the book," and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book."
5. Show that $A \oplus B = (A \cup B) - (A \cap B)$.

   Show that $A \oplus B = (A - B) \cup (B - A)$.

   Show that if A is a subset of a universal set U, then

   a) $A \oplus A = \emptyset$. **b)** $A \oplus \emptyset = A$.
   c) $A \oplus U = \overline{A}$. **d)** $A \oplus \overline{A} = U$.

   Show that if A and B are sets, then

   a) $A \oplus B = B \oplus A$. **b)** $(A \oplus B) \oplus B = A$.

6. Find the solution to each of these recurrence relations and initial conditions. Use an iterative approach such as that used in Example 10.

   a) $a_n = 3a_{n-1}, a_0 = 2$
   b) $a_n = a_{n-1} + 2, a_0 = 3$
   c) $a_n = a_{n-1} + n, a_0 = 1$
   d) $a_n = a_{n-1} + 2n + 3, a_0 = 4$
   e) $a_n = 2a_{n-1} - 1, a_0 = 1$
   f) $a_n = 3a_{n-1} + 1, a_0 = 1$
   g) $a_n = na_{n-1}, a_0 = 5$
   h) $a_n = 2na_{n-1}, a_0 = 1$

7.

Find each of these values.

a) $(177 \bmod 31 + 270 \bmod 31) \bmod 31$

b) $(177 \bmod 31 \cdot 270 \bmod 31) \bmod 31$

Find each of these values.

a) $(-133 \bmod 23 + 261 \bmod 23) \bmod 23$

b) $(457 \bmod 23 \cdot 182 \bmod 23) \bmod 23$

Find each of these values.

a) $(19^2 \bmod 41) \bmod 9$

b) $(32^3 \bmod 13)^2 \bmod 11$

c) $(7^3 \bmod 23)^2 \bmod 31$

d) $(21^2 \bmod 15)^3 \bmod 22$

Find each of these values.

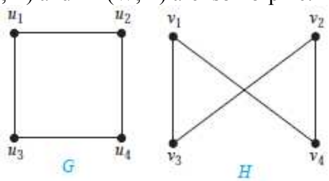
a) $(99^2 \bmod 32)^3 \bmod 15$

b) $(3^4 \bmod 17)^2 \bmod 11$

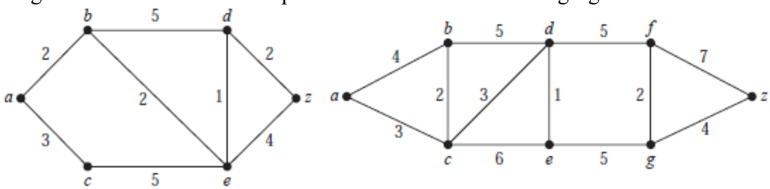
c) $(19^3 \bmod 23)^2 \bmod 31$

d) $(89^3 \bmod 79)^4 \bmod 26$

8. Write recursive algorithms for the following
   A. GCD of two number B. Factorial of an integer
   B. Fibonacci sequence D. Prime number
9. In any graph, the sum of the degrees of the vertices is twice the number of edges [or] Let $G = (V,E)$ be an undirected graph with m edges. Then $2m = \sum_{v\in V} \deg(v)$.
10. Let $G = (V,E)$ be a directed graph. Then $\sum_{v\in V} \deg^-(v) = \sum_{v\in V} \deg^+(v) = |E|$
11. Show that the graphs G=(V, E) and H=(W, F) are isomorphic.



12. Use Dijkstra algorithm to find the shortest path from ‘a’ to ‘f’ in following figures.



13. Use Huffman coding to encode these symbols with given frequencies: a: 0.20, b: 0.10, c: 0.15, d: 0.25, e: 0.30. What is the average number of bits required to encode a character?
14. Use Huffman coding to encode these symbols with given frequencies: A: 0.10, B: 0.25, C: 0.05, D: 0.15, E: 0.30, F: 0.07, G: 0.08. What is the average number of bits required to encode a symbol?
15. Write prefix and postfix expressions for $((x+y)\uparrow 2)+((x-4)/3)$
16. Write prefix and postfix expressions for $((x+2)\uparrow 3)+(y-(3+x))-5$
17. Write prefix and postfix expressions for $(x+xy)+\left(\frac{x}{y}\right)$ and $x+(\frac{xy+x}{y})$
18. Write prefix and postfix expressions for $(A\cap B)-(A\cup(B-A))$

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Mathematics with Computer Science)

MCS – 403A Semester IV

Computer Organization

Paper III (A)

Unit – I

Digital Logic Circuits: Digital Computers, Logic Gates, Boolean algebra, Map Simplification, Combinational Circuits, Flip-Flops, Sequential Circuits. Digital Components: Integrated Circuits, Decoders, Multiplexers, Registers, Shift Registers, Binary Counters, Memory Unit. Data Representation: Data Types, Complements, Fixed Point Representations, Floating Point Representation, Binary Codes, and Error Detection Codes.

Unit – II

Register Transfer and Microoperations: Register Transfer Language, Register Transfer, Bus and Memory Transfers, Arithmetic Microoperations, Logic Microoperations, and Shift Microoperations. Basic Computer Organization and Design: Instruction Codes, Computer Registers, Computer Instructions, Timing and Control, Instruction Cycle, Memory Reference Instructions, Input-Output and Interrupt, Design of Accumulator Logic. Programming the Basic Computer: Machine Language, Assembly Language, The Assembler Program Loops, Programming Arithmetic and Logic Operations, Subroutines, Input-Output Programming.

Unit – III

Central Processing Unit: Introduction, General Register Organization, Stack Organization, Instruction Formats, Addressing Modes, Data Transfer and Manipulation, Program Control, Reduced Instruction Set Computer (RISC).

Pipeline and Vector Processing: Parallel Processing, Pipelining, Arithmetic Pipelines, Instruction Pipelines and RISC Pipelines, Vector Processing. Computer Arithmetic: Addition and Subtraction, Multiplication Algorithms, Division Algorithms, and Floating Point Arithmetic Operations, Decimal Arithmetic Unit, Decimal Arithmetic Operations.

Unit – IV

Input-Output Organization: Peripheral Devices, Input-Output Interface, Asynchronous Data Transfer, Modes of Transfer, Priority Interrupt, Direct Memory Access (DMA), Input-Output Processor, Serial Communication. Memory Organization: Memory Hierarchy, Main Memory, RAM and ROM, Auxiliary Memory, Associative Memory, Cache Memory, Virtual Memory, Memory Management Hardware.

Text Books:

1. M. Morris Mano, Computer System Architecture (3e)
2. Andrew S. Tanenbaum, Structured Computer Organization
3. William Stallings, Computer Organization and Architecture
4. David A. Patterson, John L. Hennessy, Computer Organization Design
5. Sivarama P. Dandamudi, Fundamentals of Computer Organization and Design

M.Sc. (Mathematics with Computer Science)

Computer Organization

MCS 453A | Paper III (A) | Semester IV

Practical Questions

1. Write the working of 8085 simulator GNUsim8085 and basic architecture of 8085 along with small introduction.
2. Study the complete instruction set of 8085 and write the instructions in the instruction set of 8085 along with examples.
3. Write an assembly language code in GNUsim8085 to implement data transfer instruction.
4. Write an assembly language code in GNUsim8085 to store numbers in reverse order in memory location.
5. Write an assembly language code in GNUsim8085 to implement arithmetic instruction.
6. Write an assembly language code in GNUsim8085 to add two numbers using lxi instruction.
7. Write an assembly language code in GNUsim8085 to add two 8 bit numbers stored in memory and also storing the carry.
8. Write an assembly language code in GNUsim8085 to find the factorial of a number.
9. Write an assembly language code in GNUsim8085 to implement logical instructions.
10. Write an assembly language code in GNUsim8085 to implement stack and branch instructions.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Mathematics with Computer Science)

MCS – 403B Semester IV

Computer Graphics

Paper III (B)

UNIT – I

A survey of computer graphics, overview of graphic systems, Video Display devices, Raster scan systems, Random scan systems, graphic input devices, Hard copy devices. Graphics software. Output Primitives, Line Drawing Algorithms: DDA, Bressenham line Algorithm, Midpoint circle Algorithm, Ellipse Algorithm. Polygon fill Algorithms: Scan – line, Boundary fill, Flood fill Algorithms.

UNIT – II

Attributes of output primitives: Line Attributes, Curve Attributes, Area-fill and character Attributes. Two dimensional transformations: Basic transformations, homogeneous representation, composite transformation, reflection and shear transformation.

UNIT – III

Two–dimensional viewing, viewing pipeline, window to view coordinate transformation. Clipping Operations: Cohen–Sutherland line clipping, Liang–Barsky line clipping, Nicholl-Lee-Nicholl Line Clipping, Sutherland–Hodgman polygon clipping, Weiler Autherton polygon clipping.

UNIT–IV

Three dimensional object representations, polygon surfaces, polygon tables, plane Equations, cubic Bezier curves, B-spline, Octrees, 3D–transformations: Translation, Rotation, Rotation about an arbitrary point. Projections: Perspective projections and parallel projections. Visible surface detection: Back faced detection, Z-buffer Algorithm, Depth sorting Algorithm, Area subdivision Algorithm.

TEXT BOOK:

1. M. Pauline Baker, Computer Graphics, C-Version, Prentice Hall of India, second edition.
2. Computer Graphics by Harington, Mc-Graw Hill Publishing Co.

M.Sc. (Mathematics with Computer Science)

Computer Graphics

MCS 453B | Paper III (B) | Semester IV

Practical Questions

1. Line drawing algorithm DDA method
2. Line drawing algorithm Bressenham's method
3. Circle drawing algorithms – parametric and Bressenham's method
4. Eclipse drawing algorithms – parametric and Bressenham's method
5. Algorithm for polygon inside tests and testing convexity
6. Polygon filling using scan conversion method
7. Transformation on 2-D composite objects
8. Line clipping algorithms – Cohen Sutherland out code method and parametric methods.
9. Polygon clipping using Sutherland – Hodgman method
10. 3-D transformation on a cube
11. Bezier curve drawing.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Mathematics with Computer Science)

MCS – 403C Semester IV

Network Security

Paper III (C)

Unit I

Conventional encryption, Security attacks, Security, Model for network security, conventional encryption model, encryption techniques, DES, Triple DES, key distribution, random number generation.

Unit II

Public – key cryptology, principles of public – key cryptosystems, RSA algorithm, key management, distribution of public keys, public key – distribution of secret keys.

Unit III

Authentication and digital systems, authenticate requirements – functions cryptographic checksum, hash function, digital signatures authentication protocols, Kerberos, X-509 directory, authentication services Diffie – Hellmann key exchange, digital signature standards.

Unit IV

Cryptographic algorithms, The MD 5 message digest algorithm, Secure Hash algorithm, international data encryption algorithm, LUCA public key encryption – Electronic mail and management security – pretty good privacy (PGP), privacy enhanced mail.

Text Books:

1. William Stallings, Network and Internet work Security, Prentice Hall of India.

M.Sc. (Mathematics with Computer Science)

Network Security

MCS 453C — Paper III (C) — Semester IV

Practical Questions

Implementation of these algorithms using Java or C/C++

A. Encryption Techniques:
   - Polyalphabetic ciphers
   - Transposition techniques
   - Hill ciphers
   - Playfair ciphers
   - Monoalphabetic ciphers
   - DES (Data Encryption Standard) Encryption
   - Double DES
   - Triple DES

B. Public Key Cryptography using RSA
   - Key Generation
   - Encryption & Decryption Techniques
   - Diffie – Hellman Key Exchange
   - Hash Function
   - Kerberos (Client & Server) in Network
   - MD5 (Message Digest Algorithm)
   - Secure Hash Algorithm (SHA)
   - Pretty Good Privacy (PGP)
   - Authentication
   - Confidentiality
   - Cipher block chaining mode (CBC)
   - Electronic codebook mode (ECB)
   - Cipher feedback mode
   - Digital Signature Algorithms
   - Message Authentication code
   - Hash Message Authentication Code
   - Secure Multipurpose Internet mail Extension (S/MIME)
   - Enveloped data
   - Signed Data
   - Clear Signing
   - Internet Data Encryption Algorithms
   - Encryption
   - Decryption

C. Firewall Installation & Configuration on Networking OS such as Linux Server System

Departemnt of MATHEMATICS,OU
Proposed Choice Based Credit System (CBCS)
M.Sc Applied Mathematics

Semester -I

S.No.	Code No	Paper	Paper Title	Hrs/Week	Internal Assessment	Semester Exam	Total Marks	Credits
1. Core	AM 101	I	Algebra	4	20	80	100	4
2. Core	AM 102	II	Analysis	4	20	80	100	4
3. Core	AM 103	III	Mathematical Methods	4	20	80	100	4
4. Core	AM 104	IV	Mechanics	4	20	80	100	4
5. Practical	AM 151	Practical	Algebra	4	….	50	50	2
6. Practical	AM 152	Practical	Analysis	4	….	50	50	2
7. Practical	AM 153	Practical	Mathematical Methods	4	….	50	50	2
8. Practical	AM 154	Practical	Mechanics	4	….	50	50	2
			Total :	**32**				**24**

DEPARTMENT OF MATHEMATICS

OSMANIAUNIVERSITY

M.Sc.(Applied Mathematics)

Algebra

AM 101 Paper I Semester I

Unit I

Automaphisms- Conjugacy and G-sets- Normal series solvable groups- Nilpotent groups. (Pages 104 to 128 of [1])

Unit II

Structure theorems of groups: Direct product- Finitly generated abelian groups- Invariants of a finite abelian group- Sylow's theorems- Groups of orders p^2,pq . (Pages 138 to 155)

Unit III

Ideals and homomsphism- Sum and direct sum of ideals, Maximal and prime ideals- Nilpotent and nil ideals- Zorn's lemma (Pages 179 to 211).

Unit-IV

Unique factorization domains - Principal ideal domains- Euclidean domains- Polynomial rings over UFD- Rings of traction.(Pages 212 to 228)

Text Books:

[1] Basic Abstract Algebra by P.B. Bhattacharya, S.K. Jain and S.R. Nagpanl.

Reference: 1] Topics in Algebra by I.N. Herstein.

M.Sc (Applied Mathematics)

Algebra

AM 151 **Paper I** **Semester I**

Practical Questions

1. A finite group G having more than two elements and with the condition that $x^2 \neq e$ for some $x \in G$ must have nontrivialautomorphism.
2. (i) Let G be a group Define $a * x = ax,\ a, x \in G$ then the set G is a G-set

   (ii) Let G be a group Define $a * x = axa^{-1}\ a, x \in G$ then G is a G-set.
3. An abelian group G has a composition series if and only if G is finite
4. Find the number of different necklaces with p beads p prime where the beads can have any of n different colours
5. If G is a finite cyclic group of order n then the order of Aut$G$, the group of automorphisms of G, is $\phi(n)$, where ϕ is Euler's function.
6. If each element $\neq e$ of a finite group G is of order2 then $|G| = 2^n$ and $G \approx C_1 \times C_2 \times \ldots\ldots \times C_n$ where C_i are cyclic and $|C_i| = 2.$
7. (i) Show that the group $\frac{Z}{<10>}$ is a direct sum of $H = \{\overline{0}\ \overline{5}\}$ *and* $K = \{\overline{0}\ \overline{2}\ \overline{4}\ \overline{6}\ \overline{8}\}$

   (ii) Show that the group $\left(\frac{z}{\langle 4 \rangle}, +\right)$ cannot be written as the direct sum of two Subgroups of order 2.
8. (i) Find the non isomorphic abelian groups of order 360

   (ii) If a group of order p^n contains exactly one sub group each of orders $p, p^2, \_\ \_\ \_ P^{n-1}$ then it is cyclic.
9. Prove that there are no simple groups of orders 63, 56, and 36
10. Let G be a group of order 108. Show that there exists a normal subgroup of order 27 or 9.
11. (i) Let **R** be acommutative Ring wilth unity. Suppose R has no nontrivial ideals .Prove that R is a field.

    (ii) Find all ideals in Z and in $\frac{Z}{\langle 10 \rangle}$

12. (i) The only Homomorphism from the ring of integers Z to Z are the identity and Zero Mappings.
    (ii) Show that any nonzero homomorphism of a field F into a ring R is one-one.
13. For any tow ideals A and B in a Ring R (i) $\frac{A+B}{B} \approx \frac{A}{A \bigcap B}$
    (ii) $\frac{A+B}{A \cap B} \approx \frac{A+B}{A} \times \frac{A+B}{B} \approx \frac{B}{A \cap B} \times \frac{A}{A \cap B}$ In particular if $R = A + B$ then $\frac{R}{A \cap B} \approx \frac{R}{A} \times \frac{R}{B}$.
14. Let R be a commutative ring with unity in which each ideal is prime then R is a field
15. Let R be a Boolean ring then each prime ideal $P \neq R$ is maximal.
16. The commutative integral domain $R = \{a + b\sqrt{-5} / a, b \in Z\}$ is not a UFD.
17. (i) The ring of integers Z is a Euclidean domain
    (ii) The Ring of Gausion Integers $R = \{m + n\sqrt{-1} / m, n \in Z\}$ is a Euclidean domain
18. (i) Prove that $2 + \sqrt{-5}$ is irreducible but not prime in $Z(\sqrt{-5})$
    (ii) Show that $1 + 2\sqrt{-5}$ *and* 3 are relatively prime in $Z(\sqrt{-5})$
19. Let R be a Euclidean domain . Prove the following
    (i) If $b \neq 0$ *then* $\phi(a) < \phi(b)$
    (ii) If a and b are associates then $\phi(a) = \phi(b)$
    (iii) If a/b *and* $\phi(a) = \phi(b)$ *then* a and b are associates
20. Prove that every nonzero prime ideal in a Euclidean domain is maximal.

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. (AppliedMathematics)

AM – 102 Semester I

Analysis

Paper-II

Unit I

Metric spaces- Compact sets- Perfect sets- Connected sets

Unit II

Limits of functions- Continuous functions- Continuity and compactness Continuity and connectedness- Discontinuities – Monotone functions.

Unit III

Rieman- Steiltjes integral- Definition and Existence of the Integral- Properties of the integral- Integration of vector valued functions- Rectifiable waves.

Unit-IV

Sequences and series of functions: Uniform convergence- Uniform convergence and continuity- Uniform convergence and integration- Uniform convergence and differentiation- Approximation of a continuous function by a sequence of polynomials.

Text Books:

[1] Principles of Mathematical Analysis (3^{rd} Edition) (Chapters 2, 4, 6)

By Walter Rudin, Mc Graw-Hill Internation Edition

M.Sc. (Applied Mathematics)

Analysis

AM 152 Paper –II Semester -I

Practical Questions

1. Construct a bounded set of real numbers with exactly three limit points
2. Suppose E^1 is the set of all limit points of E. Prove that E^1 is closed also prove that E and E have the same limit points.
3. Let E^0 demote the set of all interior points of a set E. Prove that E^0 is the largest open set contained in E Also prove that E is open if and only if E = E^0
4. Let Xbe an infinite set.For $p \in X$, $q \in X$ define

$$d(p,q) = \begin{cases} 1 \ \ if \ p \neq q \\ 0 \ if \ \ p = q \end{cases}$$

Prove that this is a metric, which subsets of the resulting metric space are open, which areclosed? Which are compact?

5. i) If A and B are disjoint closed sets in some metric space X, prove that they are separated
   ii) Prove the same for disjoint open sets
   iii)Fixa $p \in X$ and $\delta > o$, Let A = { $q \in X : d(p,q) < \delta$}
and $B = \{q \in X : d(p,q) > \delta\}$ prove that A and B are separated.

6. i) Suppose f is a real function on R which satisfies $\lim\limits_{h \to o} [f(x+h) - f(x-h) = o]$ for every $x \in R$ Does this imply that f is continuous? Explain

   ii) Let f be a continuous real function on a metric space X,let Z(f) = $\{p \in X : f(p) = 0\}$ prove that z (f) is closed.

7. If f is a continuous mapping of a metric space X into a metric space Y .prove that

   $f(\overline{E}) \subset \overline{f(E)}$ for every set E $\subset$ X

8. Let f and g be continuous mapping of a metric space X into a metric space Y Let E be a dense subset of X. Prove that
   i) f(E) is dense in f(X)
   ii) If g(p) =f(p) $\forall$ p $\in$ E , Prove that g(p) =f(p) $\forall$ p $\in$ X

9. Suppose f is a uniformly continuous mapping of a metric space X into a metric space Y and { X_m} is a Couchy sequence in X prove that {f(X_m)} is Cauchy sequence in Y

10. Let I = [0 , 1] be the closed unit interval, suppose f is a continuous mapping of f into I. Prove that f(x) = x for at least one x

11. Suppose α increases on [a , b] , a <x_o<b, α is continuous at x_0, $f(x_0)$ = 1 and f(x) =0 if x $\neq$ x_o .Prove that f $\in$ R(α) and $\int_a^b f\,d\alpha = 0$

12. Suppose f $\geq$0 and f is continuous on [a , b] and $\int_a^b f(x)dx = 0$, Prove that f(x) =0 $\forall$ x€ [a , b]

13. If f(x) = 1or 0 according as x is rational or not .Prove that f $\notin R$ on [a , b] for any a,b,€R with a<b.Also prove that f $\notin$ R(α) on [a , b] with respect to any monotonically increasing function α on [a , b]

14. Suppose f is a bounded real function on [a , b] and f^2€R on [a , b]. Does it follow that $f \in R$?

    Does the answer change if we assume that f^3€R?

15. Suppose γ_1 *and* γ_2 are the curves in the complex plane defined on [0,2π] by γ_1(t)= e^{it} , $\gamma_2(t) = e^{2it}$

    Show that the two curves have the same range

    Also Show that γ_1 *and* γ_2 are rectifiable and find the curve length of γ_1 *and* γ_2

16. Discuss the uniform conversance of the sequence of functions {f_n} where

$$f_n(x) = \frac{\sin nx}{\sqrt{n}} \quad \text{x real, n = 1,2,3....}$$

17. Give an example of a series of continuous functions whose sum function may be discontinuous.
18. Discuss the uniform conversance of the sequence

$$f_n(x)= \frac{1}{1+nx} \; x > 0,\; n = 1,2,3...$$

19. Give an example of a sequence of functions such that

$$\lim \int f_n \neq \int \lim f_n$$

20. Prove that a sequence {f_n} converse to f with respect to the metric of C(x) if and only if $f_n \rightarrow$ f uniformly on X

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. (Mathematics)

AM – 103 Semester I

Mathematical Methods

Paper- III

Unit I

Existence and Uniqueness of solution of $\frac{dy}{dx} = f(x,y)$. The method of successive approximation- Picard's theorem- Sturm-Liouville's boundary value problem.
Partial Differential Equations: Origins of first-order PDES-Linear equation of first-order- Lagrange's method of solving PDE of P_p+Qq = R – Non-Linear PDE of order one-Charpit method- Linear PDES with constant coefficients.

Unit II

Partial Differential Equations of order two with variable coefficients- Canonical form Classification of second order PDE- separation of variable method solving the one-dimensional Heat equation and Wave equation- Laplace equation.

Unit III

Power Series solution of O.D.E. – Ordinary and Singular points- Series solution about an ordinary point -Series solution about Singular point-Frobenius Method.
Lagendre Polynomials: Lengendre's equation and its solution- Lengendre Polynomial and its properties- Generating function-Orthogonal properties- Recurrance relations- Laplace's definite integrals for P_n (x)- Rodrigue's formula.

Unit-IV

Bessels Functions: Bessel's equation and its solution- Bessel function of the first kind and its properties- Recurrence Relations- Generating function- Orthogonality properties.
Hermite Polynomials: Hermite's equation and its solution- Hermite polynomial and its properties- Generating function- Alternative expressions (Rodrigue's formula)- Orthogonality properties- Recurrence Relations.

Text Books:

[1] "Elements of Partial Differential Equations", By Ian Sneddon, Mc.Graw-Hill International Edition.
[2] "Text book of Ordinary Differential Equation", By S.G.Deo, V. Lakshmi Kantham, V. Raghavendra, Tata Mc.Graw Hill Pub. Company Ltd.
[3] "Ordinary and Partial Differential Equations", By M.D. Raisingania,
S. Chand Company Ltd., New Delhi.

M.Sc.Applied Mathematics
Mathematical Methods

AM 153 | Paper III | Semester I

Practical Questions

1. Compute the first three successive approximations for the solution of the initialvalue problem $\frac{dx}{dt} = x^2, \quad x(0) = 1.$

2. Solve $yp = 2yx + \log q.$

3. Solve $yzp + zxq = xy$ with usual notations.

4. Explain Strum-Liouille's boundary value problems.

5. Classify the equation $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0.$

6. Solve $r + t + 2s = 0$ with the usual notations.

7. Find the particular integral of the equation $(D^2 - D)Z = e^{2x+y}$.

8. Solve in series the equation $xy'' + y' - y = 0$.

9. Solve $y'' - y = x$ using power series method.

10. Solve the Froenius method $x^2 y'' + 2x^2 y' - 2y = 0$.

11. Solve in series $2xy'' + 6y' + y = 0.$

12. Prove that $J_{-n}(x) = (-1)^n J_n(x)$ where n is an integer.

13. Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$.

14. Prove that $H_n(-x) = (-1)^n H_n(x)$.

15. Show that $H_{2n+1}(0) = 0$.

16. Show that $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$.

17. Solve $\dfrac{\partial u}{\partial x} = \dfrac{\partial u}{\partial y}$; *with* $u(x,0) = 4e^{-x}$ using separation of variable method.

18. Find the surface passing through the parabolas $z = 0,\ y^2 = 4ax$ *and* $z = 1,\ y^2 = -4ax$ and satisfying the equation $xr + 2p = 0$.

19. Find the surface satisfying $t = 6x^2 y$ containing two lines y = 0 = z and y = 2 = z .

20. Reduse the equation $x^2 r - y^2 t + px - qy = x^2$ in to canonical form.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Applied Mathematics)

AM 104 Semester I

Mechanics
Paper IV

Unit I

Newton's Law of Motion: Historical Introduction, Rectilinear Motion: Uniform Acceleration Under a Constant Force, Forces that Depend on Position: The Concepts of Kinetic and Potential Energy, Dynamics of systems of Particles:- Introduction - Centre of Mass and Linear Momentum of a system- Angular momentum and Kinetic Energy of a system, Mechanics of Rigid bodies- Planar motion:- Centre of mass of Rigid body-some theorem of Static equilibrium of a Rigid body- Equilibrium in a uniform gravitational field.

Unit II

Rotation of a Rigid body about a fixed axis, Moment of Inertia:- calculation of moment of Inertia Perpendicular and Parallel axis theorem- Physical pendulum-A general theorem concerning Angular momentum-Laminar Motion of a Rigid body-Body rolling down an inclined plane (with and without slipping).

Unit III

Motion of Rigid bodies in three dimension-Angular momentum of Rigid body products of Inertia, Principles axes-Determination of principles axes-Rotational Kinetic Energy of Rigid body- Momentum of Inertia of a Rigid body about an arbitrary axis- The momental ellipsoid - Euler's equation of motion of a Rigid body.

Unit IV

Lagrange Mechanics:-Generalized Coordinates-Generalized forces-Lagrange's Equations and their applications-Generalized momentum-Ignorable coordinates-Hamilton's variational principle-Hamilton function-Hamilton's Equations- Problems-Theorems.

Text Book:

[1] G.R.Fowles, Analytical Mechanics, CBS Publishing, 1986.

M.Sc.(Applied Mathematics)

Mechanics

AM 154 Paper IV Semester I

Practical Questions

1. Discuss the motion of particle sliding down a smooth inclined plane at an angle θ to the horizontal.
2. Discuss the centre of mass of Solid homogeneous sphere of radius a.
3. Discuss the centre of mass of Hemispherical shell of radius a.
4. Discuss the centre of mass of Quadrant of uniform circular lamina of radius b.
5. Find the centre of mass of area bounded by a parabola $y=x^2/b$ and line $y=b$.
6. Point the moment of inertia of following:
   a. Rectangular lamina about a line passing through centre and normal to it,
   b. Rectangular parallelepiped,
   c. Circular wire and disk,
   d. Elliptic disk,
   e. Hollow sphere about a diameter, Solid sphere about a diameter.
7. Point the moment of inertia of a hollow sphere about diameter, its external and internal radii being a and b.
8. Find the moment of inertia of a uniform circular cylinder of length b and radius a about an axis through the centre and perpendicular to the central axis.
9. A circular hoop of radius a swing as a physical pendulum about a point on the circumference. Find the period of oscillation for small amplitude if the axis of rotation is (a) normal to the plane of the hoop and (b) in the plane of the hoop.
10. Find the acceleration of a uniform circular cylinder rolling down an inclined plane.
11. Find the direction of the principle axis in the plane of rectangular lamina of sides a and b at a corner.
12. Find the principle moments of inertia of a square plate about a corner.
13. Find the directions of principle axes for the above problem.
14. Find the inertia tensor for a square plate of side l and mass m in a coordinate system OXYZ where O is at corner and X and Y are along the two edges. Also find angular momentum and kinetic energy of rotation.
15. A thin uniform rectangular plate is of mass m and dimension 2a x a. Choose coordinate system OXYZ such that the plate lies in the XY plane with origin at the corner, the long dimension being along the X axis. Find the following:
   a. The moments and products of inertia,
   b. The moment of inertia about the diagonal through the origin,
   c. The angular momentum about the origin if the plate is spinning with angular rate w about the diagonal through the origin,
   d. The kinetic energy in part c.
16. Derive the governing equation for 1D damped harmonic oscillation.
17. Find the governing equation for single particle in central field.
18. Find the governing equation for a particle sliding on a movable inclined plane.
19. A mass suspended at the end of a light spring having spring constant k is set into vertical motion. Use the Lagrange equation to find the equation of motion.
20. Find the acceleration of a solid uniform sphere rolling down a perfectly rough fixed inclined plane.

Departemnt of MATHEMATICS,OU
Proposed Choice Based Credit System (CBCS)
M.Sc Applied Mathematics

Semester -II

S.No.	Code No	Paper	Paper Title	Hrs/Week	Internal Assessment	Semester Exam	Total Marks	Credits
1. Core	AM 201	I	Advanced Algebra	4	20	80	100	4
2. Core	AM 202	II	Advanced Analysis	4	20	80	100	4
3. Core	AM 203	III	Complex Analysis	4	20	80	100	4
4. Core	AM 204	IV	Fluid Mechanics	4	20	80	100	4
5. Practicals	AM 251	Practical	Advanced Algebra	4	….	50	50	2
6. Practicals	AM 252	Practical	Advanced Analysis	4	….	50	50	2
7. Practicals	AM 253	Practical	Complex Analysis	4	….	50	50	2
8. Practicals	AM 254	Practical	Fluid Mechanics	4	….	50	50	2
			Total :	**32**				**24**

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. (Applied Mathematics)

AM –201 Semester II

Advanced Algebra

Paper I

Unit I

Algebraic extensions of fields: Irreducible polynomials and Eisenstein criterion-Adjunction of roots- Algebraic extensions-Algebraically closed fields (Pages 281 to 299)

Unit II

Normal and separable extensions: Splitting fields- Normal extensions- Multiple roots- Finite fields- Separable extensions (Pages 300 to 321)

Unit III

Galois theory: Automorphism groups and fixed fields- Fundamental theorem of Galois theory- Fundamental theorem of Algebra (Pages 322 to 339)

Unit-IV

Applications of Galoes theory to classical problems: Roots of unity and cyclotomic polynomials- Cyclic extensions- Polynomials solvable by radicals- Ruler and Compass constructions. (Pages 340-364)

Text Books:

[1] Basic Abstract Algebra- S.K. Jain, P.B. Bhattacharya, S.R. Nagpaul.

Reference Book: Topics in Algrbra B y I. N. Herstein

M.Sc Applied mathematics

Advanced Algebra

AM 251 Paper I Semester II

Practical Questions

1. (i) $\phi_p(x) = 1 + x + ... + x^{p-1}$ is irreducible over Q. Where p is a prime.

   (ii) Show that $x^3 + 3x + 2 \in \frac{Z}{\langle 7 \rangle}(x)$ is irreducible over the field $\frac{Z}{\langle 7 \rangle}$.

2. Show that the following polynomials are irreducible over Q

   (i) $x^3 - x - 1$ (ii) $x^4 - 3x^2 + 9$ (iii) $x^4 + 8$

3. Show that there exists an extension of $E \; of \; \frac{Z}{\langle 3 \rangle}$ with nine elements having all the roots of $x^2 - x - 1 \in \frac{Z}{\langle 3 \rangle}(x)$

4. (i) Show that there is an extension E of R having all the roots of $1 + x^2$

   (ii) Let $f_i(x) \in F(x)$ for i= 1, 2, ...m then there exists an extension E of F in which each polynomial has root

5. Show that $\sqrt{2} \; and \; \sqrt{3}$ are algebraic over Q and find the degree of $Q(\sqrt{2})$ over Q and $Q(\sqrt{3})$ over Q.

   (iii) Find a suitable number a such that $Q(\sqrt{2}, \sqrt{5}) = Q(a)$.

6. Show that the degree of the extension of the splitting field of $x^3 - 2 \in Q(x)$ is 6

7. Let p be a prime then $f(x) = x^p - 1 \in Q(x)$ has a splitting field $Q(\alpha)$ where $\alpha \neq 1$ and $\alpha^p = 1$. Also $[Q(\alpha) : Q] = p - 1$

8. Show that the splitting field of $f(x) = x^4 - 2 \in Q(x) \; over \; Q \; is \, Q(2^{\frac{1}{4}}, i)$ and its degree of extension is 8

9. If the multiplicative group F^* of non zero elements of a field F is cyclic then F is Finite

10. The group of automorphisms of a field F with p^n elements is cyclic of order n and generated by ϕ where $\phi(x) = x^p$, $x \in F$

11. The group $G(\frac{Q(\alpha)}{Q})$ where $\alpha^5 = 1$ *and* $\alpha \neq 1$ is isomorphic to the cyclic group of order 4
12. Let $E = Q(\sqrt[3]{2}, \omega)$ *where* $\omega^3 = 1, \omega \neq 1$ let σ_1 be the identity automorphism of E and Let σ_2 be an automorphism of E such that $\sigma_2(\omega) = \omega^2$ and $\sigma_2(\sqrt[3]{2}) = \omega(\sqrt[3]{2})$. If $G = \{\sigma_1, \sigma_2\}$ then $E_G = Q(\sqrt[3]{2}\omega^2)$
13. If $f(x) \in F(x)$ has r distinct roots in its splitting field E over F then the Galois group $G\left(\frac{E}{F}\right)$ *of* $f(x)$ is a subgroup of the symmetric group S_r.
14. The Galois group of $x^4 - 2 \in Q(x)$ is the octic group.
15. The Galois group of $x^4 + 1 \in Q(x)$ is Klein four group
16. $\phi_8(x)$ and $x^8 - 1$ have the same Galois group namely $\left(\frac{Z}{\langle 8 \rangle}\right)^* = \{1,3,5,7\}$, *the* Klein's four group.
17. If a field F contains a primitive n^th root of unity then the characteristic of F is Zero or a prime P that does not divide n
18. Show that the following polynomials are not solvable by radicals over Q
    (i) $x^7 - 10x^5 + 15x + 5$ (ii) $x^5 - 9x + 3$ (iii) $x^5 - 4x + 2$

19. It is impossible to construct a cube with a volume equal to twice the volume of a given cube by using ruler and compass only.

20. A regular n-gon is constructible if and only if $\phi(n)$ is a power of 2. (equivalently the angle $\frac{2\pi}{n}$ is Constructible.)

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. (Applied Mathematics)

AM –202 **Semeste II**

Advanced Analysis
Paper II

Unit I

Algebra of sets- Borel sets- Outer measure- Measurable sets and Lebesgue measure- A non-measurable set- Measurable functions- Little word's three principles.

Unit II

The Rieman integral- The Lebesgue integral of a bounded function over a set of finite measure- The integral of a non-negative function- The general Lebesgue integral.

Unit III

Convergence in measure- Differentiation of a monotone functions- Functions of bounded variation.

Unit-IV

Differentiation of an integral- Absolute continuity- The L^p-spaces- The Minkowski and Holder's inequalities- Convergence and completeness.

Text Books:[1] Real Analysis (3rd Edition) (Chapters 3, 4, 5)

by

H. L. Royden

Pearson Education (Low Price Edition)

M.Sc. Applied Mathematics

Advanced Analysis

AM252 Paper II SemesterII

Practical Questions

1. i. Prove that the interval [0,1] is not countable.

   ii. If A is the set of all irrational numbers in [0,1]. Prove that $m^*(A) = 1$.

2. i. If $m^*(A) = 0$. Prove that $m^*(A \cup B) = m^*(B)$.

ii. Prove that if a σ −algebra of subsets of $\mathbb{R}$ contains intervals of the form (a, ∞) then it contains all intervals.

3. Show that a set E is measurable if and only if for each $\epsilon > 0$ there exists a closed set F and an open set O such that $F \subseteq E \subseteq O$ and $m^*(O - F) < \epsilon$.

4. i. Show that if E_1 and E_2 are measurable then $m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$

   ii. Suppose $\{A_k\}$ is an ascending collection of measurable sets. Prove that

$$m(\cup_{k=1}^{\infty} A_k) = \lim_{k\to\infty} m(A_k)$$

5. Suppose A and B are any sets. Prove that

   i. $\chi_{A\cap B} = \chi_A \chi_B$

   ii. $\chi_{A\cup B} = \chi_A + \chi_B - \chi_A \chi_B$

   iii. $\chi_{A^c} = 1 - \chi_A$

6. Let E have measure zero. Show that if f is a bounded function on E then f is measurable and $\int_E f = 0$.

7. Let $\{f_n\}$ be a sequence of non negative measurable functions that converge to f pointwise on E. Let $M \geq 0$ be such that $\int_E f_n \leq M$ for all n. Show that $\int_E f \leq M$.

8. Let f be a non negative measurable functions on E.

   Prove that $\int_E f = 0$ if and only if $f = 0$ a.e on E.

9. Let $\{f_n\}$ be a sequence of non negative measurable functions on E that converges pointwise on E to f. Suppose $f_n \leq f$ on E for each n, show that $\lim_{n\to\infty} \int_E f_n = \int_E f$.

10. Suppose $\{f_n\}$ is a sequence of measurable functions on E that converges pointwise on a.e. on E to f.

Suppose there is a sequence $\{g_n\}$ of non negative measurable functions on E that converges

pointwise on a.e. on E to g and dominates $\{f_n\}$ on E in the sense that $|f_n| \leq g_n$ on E $\forall n$.

If $\lim_{n\to\infty} \int_E g_n = \int_E g$ prove that $\lim_{n\to\infty} \int_E f_n = \int_E f$.

11. Prove that pointwise convergence implies convergence in measure.

12. Construct a sequence of measurable functions which converges in measure but not point wise.

13. Suppose f, g are functions of bounded variation in [a,b]. Show that $f + g$ and λf for any scalar λ are

also functions of bounded variation on $[a, b]$.

Also prove that i. $\tau_a^b(f + g) \leq \tau_a^b(f) + \tau_a^b(g)$

ii. $\tau_a^b(\lambda f) = |\lambda|\tau_a^b(f)$

14. Prove that the greatest integer function is a function of bounded variation on $[a, b]$

15. Show that continuous and bounded variation of a function are two independent concepts.

16. Show that the sum and difference and product of two absolutely continuous functions are also absolutely continuous.

17. Let f be absolutely continuous on $[c, d]$ and g be absolutely continuous on $[a, b]$with $c \leq g \leq d$. Prove that fog is absolutely continuous on $[a, b]$

18. Suppose f is absolutely continuous on $[a, b]$ and $E = \{x: f'(x) = 0\}$. Prove that $m\big(g(E)\big) = 0$

Note. f is absolutely continuous on E with $f'(x) = 0$ $\forall x \in E$ implies $f$is constant on E which implies $m\big(f(E)\big) = 0$

19. Let g be an absolutely continuous monotone function on [0,1] and E is a set of measure zero. Prove that $g(E)$ has measure zero.

20. i. Show that $\|f + g\|_\infty \leq \|f\|_\infty + \|g\|_\infty$

ii. If $f \in L^p$, $g \in L^p$ then prove that $f + g \in f \in L^p$

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc.Applied Mathematics

AM203 Semester II

Complex Analysis

Paper- III

UNIT-I

Regions in the Complex Plane -Functions of a Complex Variable - Mappings -Mappings by the Exponential Function- Limits - Limits Involving the Point at Infinity - Continuity -Derivatives - Cauchy–Riemann Equations -Sufficient Conditions for Differentiability - Analytic Functions - Harmonic Functions -Uniquely Determined Analytic Functions - Reflection Principle - The Exponential Function -The Logarithmic Function -Some Identities Involving Logarithms -Complex Exponents -Trigonometric Functions -Hyperbolic Functions

UNIT-II

Derivatives of Functions w(t) -Definite Integrals of Functions w(t) - Contours -Contour Integrals - Some Examples -Examples with Branch Cuts -Upper Bounds for Moduli of Contour Integrals – Anti derivatives -Cauchy–Goursat Theorem -Simply Connected Domains- Multiply Connected Domains-Cauchy Integral Formula -An Extension of the Cauchy Integral Formula -Liouville's Theorem and the Fundamental Theorem of Algebra -Maximum Modulus Principle

UNIT-III

Convergence of Sequences - Convergence of Series - Taylor Series -Laurent Series -Absolute and Uniform Convergence of Power Series- Continuity of Sums of Power Series - Integration and Differentiation of Power Series - Uniqueness of Series Representations-Isolated Singular Points - Residues -Cauchy's Residue Theorem - Residue at Infinity - The Three Types of Isolated Singular Points - Residues at Poles -Examples -Zeros of Analytic Functions -Zeros and Poles -Behavior of Functions Near Isolated Singular Points

UNIT-IV

Evaluation of Improper Integrals -Improper Integrals from Fourier Analysis - Jordan's Lemma - Indented Paths - - Definite Integrals Involving Sines and Cosines - Argument Principle -Rouché's Theorem -Linear Transformations -The Transformation w = 1/z - Mappings by 1/z -Linear Fractional Transformations -An Implicit Form -Mappings of the Upper Half Plane

Text: James Ward Brown, Ruel V Churchill, *Complex Variables with applications*

M.SC. Applied Mathematics
Complex Analysis

AM253Semester-II

Paper-III
Practical Questions

1

In each case, determine the singular points of the function and state why the function is analytic everywhere except at those points:

(a) $f(z) = \dfrac{2z+1}{z(z^2+1)}$; *(b)* $f(z) = \dfrac{z^3+i}{z^2-3z+2}$; *(c)* $f(z) = \dfrac{z^2+1}{(z+2)(z^2+2z+2)}$.

2

Show that $u(x, y)$ is harmonic in some domain and find a harmonic conjugate $v(x, y)$ when

(a) $u(x, y) = 2x(1-y)$; *(b)* $u(x, y) = 2x - x^3 + 3xy^2$;
(c) $u(x, y) = \sinh x \sin y$; *(d)* $u(x, y) = y/(x^2+y^2)$.

3

Find all values of z such that

(a) $e^z = -2$; *(b)* $e^z = 1 + \sqrt{3}i$; *(c)* $\exp(2z-1) = 1$.

4

Let the function $f(z) = u(x, y) + iv(x, y)$ be analytic in some domain D. State why the functions

$$U(x, y) = e^{u(x,y)} \cos v(x, y), \quad V(x, y) = e^{u(x,y)} \sin v(x, y)$$

are harmonic in D and why $V(x, y)$ is, in fact, a harmonic conjugate of $U(x, y)$.

5

Show that

(a) $(1+i)^i = \exp\left(-\dfrac{\pi}{4} + 2n\pi\right) \exp\left(i\dfrac{\ln 2}{2}\right)$ $(n = 0, \pm 1, \pm 2, \ldots)$;
(b) $(-1)^{1/\pi} = e^{(2n+1)i}$ $(n = 0, \pm 1, \pm 2, \ldots)$.

6

Let C denote the line segment from $z = i$ to $z = 1$. By observing that of all the points on that line segment, the midpoint is the closest to the origin, show that

$$\left| \int_C \frac{dz}{z^4} \right| \le 4\sqrt{2}$$

without evaluating the integral.

7

Show that if C is the boundary of the triangle with vertices at the points 0, $3i$, and -4, oriented in the counterclockwise direction (see Fig. 48), then

$$\left| \int_C (e^z - \bar{z})\, dz \right| \le 60.$$

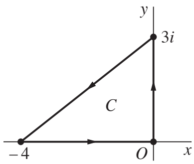


FIGURE 48

8

Let C be the unit circle $z = e^{i\theta}(-\pi \le \theta \le \pi)$. First show that for any real constant a,

$$\int_C \frac{e^{az}}{z}\,dz = 2\pi i.$$

Then write this integral in terms of θ to derive the integration formula

$$\int_0^{\pi} e^{a\cos\theta}\cos(a\sin\theta)\,d\theta = \pi.$$

9

Find the value of the integral of $g(z)$ around the circle $|z - i| = 2$ in the positive sense when

(a) $g(z) = \dfrac{1}{z^2+4}$; *(b)* $g(z) = \dfrac{1}{(z^2+4)^2}$.

10

Show that for R sufficiently large, the polynomial $P(z)$ in Theorem 2, Sec. 53, satisfies the inequality

$$|P(z)| < 2|a_n||z|^n \quad \text{whenever} \quad |z| \ge R.$$

11

Obtain the Maclaurin series representation

$$z\cosh(z^2) = \sum_{n=0}^{\infty} \frac{z^{4n+1}}{(2n)!} \qquad (|z| < \infty).$$

12

Obtain the Taylor series

$$e^z = e\sum_{n=0}^{\infty} \frac{(z-1)^n}{n!} \qquad (|z-1| < \infty)$$

for the function $f(z) = e^z$ by

13

In each case, show that any singular point of the function is a pole. Determine the order m of each pole, and find the corresponding residue B.

(a) $\dfrac{z^2+2}{z-1}$; *(b)* $\left(\dfrac{z}{2z+1}\right)^3$; *(c)* $\dfrac{\exp z}{z^2+\pi^2}$.

14

Evaluate the integral

$$\int_C \frac{\cosh \pi z}{z(z^2+1)}\,dz$$

when C is the circle $|z| = 2$, described in the positive sense.

15

Show that

(a) $\operatorname*{Res}_{z=\pi i} \dfrac{z-\sinh z}{z^2 \sinh z} = \dfrac{i}{\pi}$;

(b) $\operatorname*{Res}_{z=\pi i} \dfrac{\exp(zt)}{\sinh z} + \operatorname*{Res}_{z=-\pi i} \dfrac{\exp(zt)}{\sinh z} = -2\cos(\pi t)$

16

Evaluate

$$\int_{-\infty}^{\infty} \frac{\cos x \, dx}{(x^2+a^2)(x^2+b^2)} \quad (a > b > 0).$$

17

Derive the integration formula

$$\int_0^{\infty} \frac{\cos(ax) - \cos(bx)}{x^2} \, dx = \frac{\pi}{2}(b-a) \qquad (a \geq 0, b \geq 0).$$

Then, with the aid of the trigonometric identity $1 - \cos(2x) = 2\sin^2 x$, point out how it follows that

$$\int_0^{\infty} \frac{\sin^2 x}{x^2} \, dx = \frac{\pi}{2}.$$

18

Evaluate

$$\int_0^{\pi} \frac{\cos 2\theta \, d\theta}{1 - 2a\cos\theta + a^2} \quad (-1 < a < 1)$$

19

Suppose that a function f is analytic inside and on a positively oriented simple closed contour C and that it has no zeros on C. Show that if f has n zeros z_k $(k = 1, 2, \ldots, n)$ inside C, where each z_k is of multiplicity m_k, then

$$\int_C \frac{zf'(z)}{f(z)} \, dz = 2\pi i \sum_{k=1}^{n} m_k z_k.$$

20

Determine the number of zeros, counting multiplicities, of the polynomial

(a) $z^4 + 3z^3 + 6$; (b) $z^4 - 2z^3 + 9z^2 + z - 1$; (c) $z^5 + 3z^3 + z^2 + 1$

inside the circle $|z| = 2$.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc: (APPLIED MATHEMATICS)

AM-204 SEMESTER-II

FLUID MECHANICS

Paper IV

Unit-I:
General Orthogonal Curvilinear Coordinates: Defination **Kinematics of fluids in motion:** Real fluids and ideal fluids – velocity of a fluid at a point – Lagrangian and Eulerian methods - stream lines, path lines and streak lines – steady and unsteady flows – The velocity potential – the vorticity vector – Local and particle rates of change – Acceleration of fluid - The Equation of Continuity (Vector and Cartesian form) – conditions at a rigid boundary.

Unit-II:
Equations of Motion of Fluid: Euler's equations of motion (Vector and Cartesian form) – Lagrange's equations of Motion - Equation in one dimensional flow problems: Bernoulli's Theorem - Applications of the Bernoulli Theorem - Kelvins circulation theorem.
Motion in two dimension: velocity potential – physical meaning of velocity potential. Stream function - physical meaning of velocity stream function.

Unit-III:
Some Two Dimensional Flows: The complex potential – Irrotational motion – stream function – Source, Sinks and Doublets and their Images – General theory of Irrotational – Milne Thomson Circle Theorem – Applications of circle theorem. The Magnus effect – The Therorem of Blasius.

Unit-IV:
Irrotational Motion in Two Dimensions: Two-dimensional irrotational motion produced by motion of circular cylinder , two coaxial cylinders. Equations of motion of a circular cylinder-Ellliptic coordinate - Motion of an Elliptic cylinder – Thoerem of Kutta-Joukowski - **Irrotational Motion in three dimensions**: Motion of a sphere through a liquid at rest at infinity – Liquid streaming past a fixed sphere – Equations of motion of a sphere

Text Books:

1. FRANK CHORLTON, "**Textbook of Fluid Dynamics**" , CBS-Publishers, New Delhi, India.
2. W.H.BESANT and A.S.RAMSEY, "**A Treatise on Hydro-Mechanics** (Part-II)", CBS-Publishers, New Delhi, India.
3. S.W.YUAN, "**Foundation on Fluid Mechanics**", Prentice-Hall India Ltd. NewDelhi.
4. M.D.RAISINGHANIA, "**Fluid Dynamics**" S.Chand & Company, New

M.Sc.(Applied Mathematics)

FLUID MECHANICS

AM 254 **Paper IV** **Semester II**

Practical Questions

PRACTICALS:

1. Find Arc length, Gradient and Divergence in Orthogonal coordinates.
2. The Laplacian and Curl operators in Orthogonal Coordinates.
3. The Cylindrical and Spherical coordinates in Orthogonal Coordinates.
4. For two-dimensional flow the velocity components are given in Eulerian system by $u = A(x+y)+Bt$ and $v = C(x+y)+Dt$, where A,B,C and D are constants. Find the displacement of a fluid particles in the Lagrangian system.
5. Show that $\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1$ is a possible form for the bounding surface of a liquid, and find an expression for the normal velocity.
6. The quantity of liquid occupies a length $2l$ of a straight tube of uniform small bore under the action of a force to a point in the tube varying as the distance from that point. It is required to determine the motion and the pressure.
7. In a region bounded by a fixed quadrantal arc and its radii, deduce the motion due to a source and an equal sink situated at the ends of one of the bounding radii. Show that the stream line leaving either end at an angle α with the radius is $r^2 \sin(\alpha+\theta) = a^2 \sin(\alpha-\theta)$.
8. At the point in an incompressible fluid having spherical polar coordinates (r, θ, ψ), the velocity components are $[2Mr^{-3}\cos\theta, Mr^{-2}\sin\theta, 0]$, where M is constant. Show that the velocity is of the potential kind. Find the velocity potential and the equations of the streamlines.
9. Test whether the motion specified by $q = \frac{k^2(x\boldsymbol{j}-y\boldsymbol{i})}{x^2+y^2}$ (k=const.) is a possible motion for an incompressible fluid. If so, determine the equations of the streamlines. Also test whether the motion is of the potential kind and if so determine the velocity potential.
10. For an incompressible fluid, $\boldsymbol{q} = [-\omega y, \omega x, 0]$ (ω=const.). Discuss the nature of the flow.
11. Bernoulli's Equation – The Pitot tube.
12. Bernoulli's Equation – The Venturi tube.
13. Find the equations of the streamlines due to uniform line sources of strength m through the points $A(-c, 0), B(c, 0)$ and a uniform line sink of strength $2m$ through the origin.
14. Describe the irrotational motion of an incompressible liquid for which the complex potential is $\omega = \boldsymbol{i}\kappa \log z$.

Applications of the Circle Theorem:

15. Obtain the complex potential for Image of a line source in a circular cylinder
16. Obtain the complex potential for Uniform flow past a stationary cylinder.

17. A circular cylinder is placed in a uniform stream, find the forces acting on the cylinder.
18. A source and sink of equal strength are placed at the points $(\pm\frac{1}{2}a, 0)$ within a fixed circular boundary $x^2 + y^2 = a^2$. Show that the streamlines are given by $\left(r^2 - \frac{1}{4}a^2\right)(r^2 - 4a^2) - 4a^2y^2 = Ky(r^2 - a^2)$.
19. The space between two infinitely long coaxial of radii **a** and **b** (**b>a**) respectively, is filled with homogeneous liquid of density ρ. The inner cylinder is suddenly moved with velocity U perpendicular to the axis, the outlet one being kept fixed. Show that the resultant impulsive pressure on a length ***l*** of the inner cylinder is

$$\pi\rho a^2 l\left(\frac{b^2 + a^2}{b^2 - a^2}\right)\mathrm{U}$$

20. A circular cylinder of radius a is moving with velocity U along the axis of x. Show that the motion produced by the cylinder in a mass of fluid at rest is given by the complex function $w = \dfrac{a^2U}{(z - Ut)}$

Departemnt of MATHEMATICS,OU
Proposed Choice Based Credit System (CBCS)
M.Sc Applied Mathematics

Semester -III

S.No.	Code No	Paper	Paper Title	Hrs/Week	Internal Assessment	Semester Exam	Total Marks	Credits
1. Core/Common	AM 301	I	Viscous Flows	4	20	80	100	4
2. Core/Common	AM 302	II	Finite Difference Methods	4	20	80	100	4
3. Elective	AM 303(A) AM 303(B) AM 303(C)	III(A) III(B) III(C)	Compressible Flows Integral Transforms Differential Geometry	4	20	80	100	4
4.Elective	AM 304(A) AM 304(B) AM 304 (C)	IV(A) IV(B) IV(C)	Operations Research Numerical Techniques Dynamical Systems	4	20	80	100	4
5. Practicals	AM 351	Practical	Viscous Flows	4	….	50	50	2
6. Practicals	AM 352	Practical	Finite Difference Methods	4	….	50	50	2
7. Practicals	AM 353(A) AM 353 (B) AM 353 (C)	Practical	Compressible Flows Integral Transforms Differential Geometry	4	….	50	50	2
8. Practicals	AM 354(A) AM 354 (B) AM 354 (C)	Practical	Operations Research Numerical Techniques Dynamical Systems	4	….	50	50	2
			Total :	**32**	**80**	**520**	**625**	**24**

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY **M.Sc:**
APPLIED MATHEMATICS

AM-301 **VISCOUS FLOW** SEMESTER-III Paper I

Unit-I:

Vortex motion: Vorticity – Kelvin's proof – Helmholtz's Vorticity Theorem – Rectilinear vortices – Two Vortex Filaments – Vortex pair – Vortex Doublet – Rectilinear Vortices - Rectilinear Vortex with circular section.

Unit-II:

Viscosity – measurement of viscosity – Stress components in a real fluid – Relations between Cartesian components of stress – Translation motion of fluid element – Stress analysis in fluid motion – Relations between stress and Rate of strain –The coefficient of viscosity and lalminar flow – The Navier-Stoke's Equation of motion of a viscous fluid.

Unit-III:

Steady motion between two parallel plates – Plane Poiseuille,Couette flows between two parallel plates – Flow through a circular pipe –The Hagen Poiseuille flow - Uniqueness Theorem - Steady motion in Tubes of uniform cross section of Elliptical and Equilateral triangle – Unsteady flow over a flat plate.

Unit-IV:

Dimensional Analysis in Fluid Mechanics –Buckingham π-Theorem – Similar flows – Reynolds number – Boundary Layer Theory – Prandtl's Boundary Layer Theory – Boundary Layer thickness –Displacement thickness – Momentum thickness-Energy thickness - Boundary Layer equations in two dimensions – The Boundary Layer along a flat plate. The Blasius solution – Approximate solutions of Boundary Layer Equations - Von Karman's Integral relation - Von Karman Integral relation by momentum law.

Text Books:

1. FRANK CHORLTON, "**Textbook of Fluid Dynamics**" , CBS-Publishers, New Delhi, India.
2. W.H.BESANT and A.S.RAMSEY, "**A Treatise on Hydro-Mechanics**(Part-II)", CBS-Publishers, New Delhi, India.
3. S.W.YUAN, "**Foundation on Fluid Mechanics**", Prentice-Hall India Ltd. NewDelhi.M.D.RAISINGHANIA, "**Fluid Dynamics**" S.Chand& Company, NewDelhi.

M.Sc. (Applied Mathematics)

AM351 **Viscous Flows** Semester –III

Paper –I

Practical Questions

1. If $u = \dfrac{ax - by}{x^2 + y^2}$, and $w = 0$, investigate the nature of the motion of liquid.
2. When an infinite liquid contains two parallel equal and opposite rectilinear vortices at a distance $2b$. Prove that the stream lines relative to the vortices are given by the equation $\log \frac{x^2+(y-b)^2}{x^2+(y+b)^2} + \frac{y}{b} = C$ the origin being the middle point of the join, which is taken for axis y.
3. Discuss the image of a vortex filament in a plane.
4. Discuss the vortex inside an infinite circular cylinder.
5. Discuss vortex outside a circular cylinder.
6. The stress tensor at a point $P$is $\sigma_{ij} = \begin{pmatrix} 7 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{pmatrix}$. Determine the stress vector on the plane at P whose unit normal is $\hat{n} = \frac{2}{3}i - \frac{2}{3}j + \frac{1}{3}k$.
7. Determine the principal stress and principal directions for the stress tensor $\sigma_{ij} = \begin{pmatrix} 6 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$.
8. Obtain the principal stresses and the corresponding principal direction for the stress tensor at a point in the fluid given by$\begin{pmatrix} 5 & 2 & 2 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$.
9. Show that the state of stress at a point is completely known if the nine components of stress tensor at that point are known.
10. Show that the stress at a point is a symmetric tensor of order two.
11. Obtain the Navier-Stokes equation in cylindrical and spherical coordinates.
12. Show that the Navier-Stokes equation for steady, viscous incompressible flow under conservative body forces may be developed in the form
$$\vec{q} \times \zeta = \nabla \left(\Omega + \frac{1}{2} q^2 + \frac{p}{\rho} \right) + \upsilon \, curl \, \zeta$$
Where$\zeta = curl \, \vec{q}$.
13. Obtain the velocity distribution for unsteady flow of a viscous incompressible fluid over an oscillating plate.
14. Prove that $\left(\nu \nabla^2 - \frac{\partial}{\partial t} \right) \nabla^2 \psi = \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x,y)}$ where ψ is the stream function for a two-dimensional motion of a viscous fluid.
15. Determine the maximum velocity, average velocity, shear stress and co-efficient of skin-friction for the Hagen-Poiseuille flow.

16. Prove that the discharge over a spill way is given by the relation $Q = VD^2 f\left(\frac{\sqrt{gD}}{V}, \frac{H}{D}\right)$. Where V- velocity of the flow; D- depth of the throat, H- Heat of water, g- acceleration due to the gravity.
17. Determine the shearing stress and boundary layer thickness, for the boundary layer flow over a flat plate.
18. Obtain Vonkarman integral equation for steady flow under no pressure gradient.
19. Application of Vonkarman's integral equation.
20. Some dimensionless numbers and its applications.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY

M.Sc. (Mathematics)

AM 302 Semester III

Finite Difference Methods

Paper II

Unit I

Partial differential Equations – Introduction - Difference method - Routh Hurwitz criterion - Domain of Dependence of Hyperbolic Equations. (1.1 to 1.4)

Unit II

Difference methods for parabolic partial differential equations - Introduction – One space dimension - two space dimensions - Spherical and cylindrical coordinate System.(2.1 to 2.3, 2.5)

Unit III

Difference methods for Hyperbolic partial differential equations - Introduction - one space dimensions - two space dimensions - First order equations.(3.1 to 3.4)

Unit IV

Numerical methods for elliptic partial differential equations – Introduction - Difference methods for linear boundary value problems - General second order linear equation - Equation in polar coordinates.(4.1 to 4.4)

Text Book:

[1] M. K. Jain, S. R. K. Iyengar, R. K. Jain,
Computational Methods for Partial Differential Equations, Wiley Eastern Limited, New Age International Limited, New Delhi.

M.Sc.(Mathematics)
Finite Diference Method

AM 352 Paper III C Semester III

Practical Questions

1. Classify the PDE $\frac{\partial^2 u}{\partial x^2} + 2x\frac{\partial^2 u}{\partial x \partial y} + (1-y^2)\frac{\partial^2 u}{\partial y^2} = 0$.
2. Classify the PDE $u_{tt}+4u_{tx}+4u_{xx}+2u_x-u_t=0$ and find its characteristics. Reduce the equation to its standard form.
3. Classify the PDE and the find the characteristics of $\frac{\partial^2 u}{\partial t^2} + (5+2x^2)\frac{\partial^2 u}{\partial x \partial t} + (1+x^2)(4+x^2)\frac{\partial^2 u}{\partial x^2} = 0$.
4. In which part of the (x, y) plane is the following equation elliptic $u_{xx}+4u_{xy}+(x^2+4y^2)u_{yy} = \sin xy$.
5. Classify and calculate the characteristics of $u_{xx}-t^2u_{tt}=0$.
6. Solve the heat conduction equation $u_t=u_{xx}$ subject to the initial and boundary conditions $u(x,0)=\sin\pi x$, $0\le x\le 1$, $u(0,t)=u(1,t)=0$ using
   a. The Schmidt method
   b. Crank-Nicolson method
7. Solve the heat conduction equation $u_t=u_{xx}$ subject to the initial and boundary conditions $u(x,0)=\sin\pi x$, $0\le x\le 1$, $u(0,t)=u(1,t)=0$, $t>0$ using
   a. Laasonen method
   b. Dufort-Frankel method
8. Use Crank-Nicolson method and central differences for the boundary conditions to solve the IVP $u_t=u_{xx}$, $u(x,0)=1$, $0\le x\le 1$, $u_t(0,t)=u(0,t)$, $u_t(1,t)= -u(1,t)$, $t>0$, with steplength $h=1/3$ and $\lambda=1/3$. Integrate upto 2 times levels.
9. Find the solution of 2D heat conduction equation $u_t=u_{xx}+u_{yy}$ subject to initial condition $u(x,y,0)=\sin\pi x \sin\pi y$, $0\le x,y\le 1$, and the boundary conditions $u=0$, on the boundaries, $t\ge 1$, using explicit method with $h=1/3$ and $\lambda=1/8$.
10. Find the solution of 2D heat conduction equation $u_t=u_{xx}+u_{yy}$ subject to initial condition $u(x,y,0)=\sin\pi x \sin\pi y$, $0\le x,y\le 1$, and the boundary conditions $u=0$, on the boundaries, $t\ge 0$, using the Peaceman-Rachford ADI method. Assume $h=1/4$, $\lambda=1/8$ and integrate for one time-step.
11. Find the solution of the IBVP $u_{tt}=u_{xx}$, $0\le x\le 1$, subject to the initial and boundary conditions $u(x,0)=\sin\pi x$, $0\le x\le 1$, $u_t(x,0)=0$, and the boundary condition $u(0,t)=u(1,t)=0$, $t>0$, using Explicit scheme
12. Find the solution of the IBVP $u_{tt}=u_{xx}$, $0\le x\le 1$, subject to the initial and boundary conditions $u(x,0)=\sin\pi x$, $0\le x\le 1$, $u_t(x,0)=0$, and the boundary condition $u(0,t)=u(1,t)=0$, $t>0$, using Implicit scheme
13. Solve the IBVP equation $u_{tt}=u_{xx}+u_{yy}$ subject to initial condition $u(x,y,0)=\sin\pi x \sin\pi y$, $u_t(x,y,0)=0$, $0\le x,y\le 1$, $u(x,y,t)=0$, on the boundary, $t\ge 0$, using the D'yakonov split form with $\theta=1/2$. Assume $h=1/3$ and $r=1/3$. Perform the integration for one time step.

14. Find the solution of $u_t+u_x=0$ subject to the initial condition

$$u(x,0) = \begin{cases} 0, & x<0 \\ x, & 0\le x\le 1 \\ 2-x, & 1\le x\le 2 \\ 0, & x>2 \end{cases}$$

using the Lax-Wendroff formula with h=1/2 and r=1/2. Compute upto 2 time step.

15. Solve $u_t+u_x=0$ subject to the initial condition $u(0,x)=\sin\pi x$, $0\le x\le 1$, using diffusion difference scheme.
16. Find the solution of $u_{xx}+u_{yy}=0$ in R subject to Dirichlet condition $u(x,y)=x-y$ on ∂R, where R is the region inside the triangle with vertices (0,0), (7,0), (0,7) and ∂R is its boundary. Assume the step length h=2.
17. Solve the equation $u_{xx}+u_{yy}= -10(x^2+y^2+10)$ over the square region with sides x=0, y=0, x=3, y=3 with u=0 on the boundary and mesh length equal to one.
18. Solve the mixed boundary value problem

$$\begin{array}{ll} u_{xx}+u_{yy}=0, & 0\le x,y\le 1 \\ u=2x, & 0\le x\le 1,\ y=0 \\ u=2x-1, & 0\le x\le 1,\ y=1 \\ u_x+u=2-y, & x=0, 0\le y\le 1 \\ u=2-y, & x=1, 0\le y\le 1 \end{array}$$

The analytical solution is $u(x,y)=2x-y$. Use five point formula with h=k=1/3.

19. Solve the mixed boundary value problem

$$\begin{array}{ll} u_{xx}+u_{yy}=0, & 0\le x^2+y^2\le 1,\ x\ge 0,\ y\ge 0 \\ u=0, & x=0,\ y=0 \\ u_n=x-y, & x^2+y^2=1 \end{array}$$

Use five point formula with h=1/2.

20. Solve the BVP $u_{rr}+(1/r)u_r+u_{zz}= -1$, $0\le r<1$, $-1<z<1$, and u=0 on the boundary using the five point scheme with h=k=1/2.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc (Applied Mathematics)

AM 303(A) **Semester III**

Compressible Flows
Paper III(A)

Unit-I

Thermodynamics and Physical properties of Gases: Introduction to equation of state – Perfect gas – First law of Thermodynamic – Internal Energy and Enthalpy, Specific Heats – Entropy and Second law of Thermodynamics and perfect gas mixture – Dissociation and Ionization – Real gases – Physical properties of gases.

Unit II

Fundamental equations of the aerodynamics of a compressible inviscid and non-heat conducting fluid – Equation of State – Equation of Continuity – Equation of motion – Equation of energy.

Unit III

Maxwell's thermodynamics relations – Isothermal, Adiabatic and Isentropic processes - Kelvin's theorem – Two dimensional flow – irrotational motion – Vortex motion – Helmholtz's theorem.

Unite IV

One dimensional flow of an inviscid compressible Fluid: Energy Equation – Velocity of sound and Mach number – Subsonic, Sonic and Supersonic - Steady flow in a Nozzle – Pressure and velocity relation in isentropic flow – Non-steady one dimensional flow – Sound wave with finite amplitude – Formation of a Shock.

Text Books:

[1] S.I,Pai, Introduction to Theory of Compressible Flow, Van Nostrand Reinhold Company.

[2] F.Chorlton, Text Book of Fluid Dynamics, CBS Publications and Distributors, New Delhi.

M.Sc. (Applied Mathematics)

AM353(A) **Compressible Flows** Semester –III

Paper –III(A)

Practical Questions

1. A simple substance is such that the internal energy u, the pressure p and the volume v of unit mass are related by $u = 3pv$. Find the specific heat at constant volume C_v and specific heat C_p. Show that $p^{3v}v^4 = F(T)$. Where $F(T)$ is a function of temperature T only.
2. For the pressure P, volume V, temperature T, internal energy U and entropy S of a given mass of fluid in equilibrium. Prove that if T and V are taken as independent variables, then
$\frac{\partial U}{\partial T} = T\frac{\partial S}{\partial T}$; $\frac{\partial U}{\partial V} = T\frac{\partial S}{\partial V} - P = T\frac{\partial P}{\partial T} - P$.
3. Obtain equation of motion of a perfect gas. For steady irrotational flow under a conservative force – grad$V$, show that
$$\int \frac{dp}{\rho} + \frac{1}{2}q^2 + V = C$$
Where $p = p(e)$ and C is a constant.
4. Obtain the general equation of motion of steady irrotationalhomentropic compressible flow in the form $a^2\nabla^2\phi + \vec{q}.\nabla\left(\frac{1}{2}q^2\right) = 0$.Where ϕ is the velocity potential, $q = -\nabla\phi$ is the fluid velocity and a is the local speed of sound in the fluid.
5. Obtain maximum mass flow through a Nozzle.
6. A gas flows adiabatically under no body forces in a thin tube whose cross-section is σ at any point of a distance x from a fixed cross-section. Show that the speed $\vec{q}$ of the gas satisfied the equation
$$\frac{d}{dx}(\log\sigma) + \left(1 - \frac{q^2}{c^2}\right)\frac{d}{dx}(\log\vec{q}) = 0$$
Where c is the speed of sound.
7. Calculate the Mach number at a point on a jet propelled aircraft, which is flying at $1100\ km/hour$ at sea level where air temperature is 20°, given that $\gamma = 1.4$ and $R = 287\ J/kg - K$.
8. Find the profile $\phi(x,t)$ of a one dimensional wave propagation if at $t = 0$; $\phi = F(x)$and $\frac{\partial\phi}{\partial t} = G(x)$.

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. (Applied Mathematics)

AM –303(B) Semester-III

Integral Transforms

Paper- III B

UNIT –I

Laplace Transforms-Existence theorem-Laplace transforms of derivatives and integrals – shifting theorems- Transform of elementary functions-Inverse Transformations-Convolution theorem-Applications to ordinary and Partial differential equations.

UNIT-II

Fourier Transforms- Sine and cosine transforms-Inverse Fourier Transforms(Infinite and Finite Transforms)-Applications to ordinary and Partial differential equations .

UNIT-III

Hankel Transforms- Hankel Transform of the derivatives of a function.- Application of Hankel Transforms in boundary value problems-The finite Hankel Transofrm.

UNIT-IV

Mellin Transforms-The Mellin inversion theorem- some elementary properties of Mellin Transforms and Mellin Transfroms of derivatives – Mellin Integrals- Convolution Theorem.

Text Books:-

1). R.V.Churchill, “Operational Mathematics”.

2). A.R.Vasishta and R.K.Guptha, “Integral Transforms”

M.Sc. (Applied Mathematics)

Integral Transforms

AM 353(B) **Paper III(B)** **Semester III**

Practical Questions

1. Show that the Laplace transform of the function $F(t)=t^n, -1<n<0,$ exists although it is not a function of class A.

2. Find the Laplace Transfarms of the function F(t), where $F(t)=\begin{Bmatrix} 2t & 0\le t\le 5 \\ 1 & t>5 \end{Bmatrix}$

3. Find $L[e^{-t}(3\sinh 2t-5\cosh 2t)].$

4. Given $L[F(t)]=\dfrac{p^2-p+1}{(2p+1)^2(p-1)}$; applying the change of scale property show that $L[F(2t)]=\dfrac{p^2-2p+4}{4(p+1)^2(p-2)}.$

5. Prove that $L\left[\dfrac{\sin t}{t}\right]=\tan^{-1}\left(\dfrac{1}{p}\right)$ *and hence find* $L\left[\dfrac{\sin at}{t}\right]$.Does the Laplace transform of $\dfrac{\cos at}{t}$ exists.

6. Find the Fourier transform of F(x) defined by $F(x)=\begin{Bmatrix} 1; & |x|<a \\ 0; & |x|>a \end{Bmatrix}$ and hence evaluate

   $(a)\ \displaystyle\int_{-\infty}^{\infty}\frac{\sin pa\cos px}{p}dp \quad and \quad (b)\int_{0}^{\infty}\frac{\sin p}{p}dp$.

7. Apply Fourier cosine transform of $f(x)=\dfrac{1}{1+x^2}$ and hence find Fourier sine transform of $F(x)=\dfrac{x}{1+x^2}.$

8. Solve for F(x) the integral equation $\displaystyle\int_0^{\infty}F(x)\sin xt\,dx=\begin{Bmatrix} 1, & 0\le t<1 \\ 2, & 1\le t<2 \\ 0, & t\ge 2 \end{Bmatrix}$

9. Show that the finite sine transform of $\dfrac{x}{\pi}$ *is* $(-1)^{p+1}\dfrac{1}{p}$.

10. Solve

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\quad x > 0,\ t > 0\, subject\, to\, the\, condition\, (i)\quad u = 0,\ where\ x = 0,\ t > 0$$

$$(ii)\ u = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}\ when\, t = 0\ and$$

$$(iii)\ u(x,t)\, is\ bounded.$$

11. Find the Hankel Transform of $x^{-2}e^{-x}$; *taking* $xJ_1(px)$ as the kernel.

12. Find the Hankel Transform of $f(x) = \begin{cases} x^n, & 0 < x < a \\ 0\ , & x > a \end{cases}$ $n > -1$ *taking* $xJ_n(px)$ as the kernel.

13. Find the Hankel transfom of $\dfrac{d^2 f}{dx^2} + \dfrac{1}{x}\dfrac{df}{dx} - \dfrac{n^2}{x^2} f.$

14. Show that $\int_0^a r^3 J_0(pr)dr = \dfrac{a^2}{p^2}[2J_0(pa) + (ap - \dfrac{4}{ap})J_1(pa)].$

15. Find the Hankel transform of $\dfrac{d^2 f}{dx^2} + \dfrac{1}{x}\dfrac{df}{dx}$ where p is the root of the equation $J_n(ap) = 0$

16. Prove that $M[f(ax);p] = a^{-p} f^*(p).$

17. Find $M[(1+x)^{-2}].$

18. Prove that $M[1+x^a]^{-\alpha} = \dfrac{\Gamma(p/a)\Gamma(\alpha - p/a)}{a\Gamma(\alpha)}$, $0 < \operatorname{Re} P < \operatorname{Re}(a\alpha).$

19. Prove that $M[(x\dfrac{d}{dx})f(x);p] = -pf^*(p).$

20. Find the Mellin transform of cosx.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc.Applied Mathematics

AM –303(C) Semester III

Differential Geometry

Paper-III(c)

Unit I

Space Curves, Tangent Line, Contact of order of a curve and a surface, Osculating Plane, Principal normal, Binormal, Torsion-Curvature-Serret-Frenet formulae-Examples thereon, The Osculating Circle-Osculating Sphere-Helices Involutes and Evolutes-Examples thereon.

Unit II

CurvesonSurfaces tangent plane-Normal, Parametric curves, First order magnitudes-Second order magnitudes-Direction coefficients-Double family of curves, Curvature of normal section-Meunier's theorem-Examples thereon.

Unit III

Principal directions and curvatures-First curvatures Gaussian curvatures, Euler's theorem. The surface z=f(x,y), Surface of revolution-Examples thereon, Geodesics, Normal property of Geodesics-Geodesics curvature, Torsion-Joachimsthal's theorem.

Unit IV

Envelops characteristics-Edge of regression-Developable surfaces-Osculating developable-Polar developable-Rectifying developable,Envelopes-Characteristic points-Examples thereon.

Text Book:

[1] C.E. Weatherburn, Differential Geometry of three dimensions, (E.L.B.S.Edition,1964).

Reference Books:

[2] T.J. Willmore, An Introduction to differential geometry(Oxford University press), 11th Edition, New Delhi,1993.

[3] Mittal and Agarwal, Differential Geometry(Krishna Prakashan Media (P) Ltd.) 12th Edition.

M.Sc. (Applied Mathematics)

AM353(c) Differential Geometry Semester –III

Paper –III

Practical Questions

(1) Define contact of nth order of a curve and a surface. Prove that if the circle $lx+my+nz=0$, $x^2+y^2+z^2=cz$ has three pt contact with paraboloid $ax^2+by^2=2z$ then $C=\frac{(l^2+m^2)}{(bl^2+cm^2)}$

(2) State and prove Serret-Frenet formulae. Prove that $[\mathbf{t}^{I},\mathbf{t}^{II},\mathbf{t}^{III}]=k^5\left(\frac{\tau}{k}\right)'$

(3) For the given curve $\mathbf{r}=(e^{-u}\sin u, e^{-u}\cos u, e^{-u})$. Find at any point of this curve(i) unit tangent vector **t**, (ii) the equation of tangent(iii). The equation of normal plane, (iv) the unit principal normal **n**, (v) the curvature, (vi) the equation of principal normal, (vii) the unit binormal vector **b**, (viii) the equation of binormal.

(4) Find the centre and radius of Osculating Sphere. Prove that the curve given by $x=a\sin u$, $y=0$, $z=a\cos u$ lies on a sphere.

(5) Show that a necessary and sufficient condition for a curve be helix is that the ratio of the curvature and torsian is constant. Find the equation involute and find its curvature

(6) Define two fundamental forms. Determine the unit-normal and the fundamental forms of the surfaces.

$r=(u\cos v, u\sin v, f(v))$

(7) Calculate the fundamental magnitudes for the Monge's form of the surface $z=f(x,y)$. Calculate the fundamental magnitudes and normal to the surface $2z=ax^2+2hxy+by^2$ taking x,y as parameters.

(8) Define Direction coefficients and ratios. Find the condition that the two families represented by the equation $Pdu^2+2Qdudv+Rdv^2=0$ are orthogonal if and only if $ER-2FQ+GP=0$ by using tangent to these directions.

(9) Show that the curves $du^2-(u^2+a^2)dv^2=0$ form an orthogonal system on the right helicoid $\mathbf{r}=(u\cos v, u\sin v, av)$ and show that on a right helicoid, the family of curves orthogonal to the curves $u\cos v=\text{constant}$ is the family $(u^2+a^2)\sin^2 v=\text{constant}$.

(10) Define normal curvature . Show that the normal to the surface

$x=(u+v)/\sqrt{2}$, $y=(u-v)/\sqrt{2}$, $z=uv$ at a point (u,v) is
$\mathbf{n}=(x,-y,-1)/\sqrt{(1+x^2+y^2)}=(u+v,\ v-u,-\sqrt{2})/(\sqrt{2})\sqrt{(1+u^2+v^2)}$ also evaluate curvature of normal section.

(11) Define principal directions and principal curvatures and Derive the equations.

(12) Show that the principal radii of curvature of the surface $y\cos(z/a)=x\sin(z/a)$ are equal to $\pm(x^2+y^2+a^2)/a$ and find principal directions(Lines of Curvature).

(13) Define Gaussian curvature and find the Gaussian curvature at any point of the right helicoid $\mathbf{r}=(u\cos v,\ u\sin v,\ av)$. Hence show that a right helicoid is a minimal surface.

(14) Find the Geodesics on a surface of Revolution $(u\cos v,\ u\sin v,\ f(u))$.

(15) Determine curvature and torsion of geodesics. If k, τ are curvature and torsion of a Geodesic, Prove that $\tau^2=(k-k_a)(k_b-k)$.

(16) Define envelope and write its equation.If a plane makes intercepts a,b,c on the axes, so that $1/a^2+1/b^2+1/c^2=1/k^2$, show that its envelopes is a conicoid which has the axes as equal conjugate diameters.

(17) Define edge of regression and find equation of edge of regression of the envelope. A sphere of constant radius b have their centres on the fixed circle $x^2+y^2=a^2$, $z=0$. Prove that their envelope is the surface

$(x^2+y^2+z^2+a^2-b^2)=4a^2(x^2+y^2)$.

(18) Find the condition that $z=f(x,y)$ may represents a developable surface. Prove that surface $xy=(z-c)^2$ is developable.

(19) Prove that curve itself is the edge of regression of the osculating developable and also prove that the edge of regression of envelope of the normal planes is the locus of the centre of osculating spheres.

(20) Find the equation of the developable surface which passes through the curves

$z=0$, $y^2=4ax$, $x=0$, $y^2=4bz$

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Applied Mathematics)

AM – 304 A Semester III

Operations Research

Paper IV(A)

Unit I

Formulation of Linear Programming problems, Graphical solution of Linear Programming problem, General formulation of Linear Programming problems, Standard and Matrix forms of Linear Programming problems, Simplex Method, Two-phase method, Big-M method, Method to resolve degeneracy in Linear Programming problem, Alternative optimal solutions. Solution of simultaneous equations by simplex Method, Inverse of a Matrix by simplex Method, Concept of Duality in Linear Programming, Comparison of solutions of the Dual and its primal.

Unit II

Mathematical formulation of Assignment problem, Reduction theorem, Hungarian Assignment Method, Travelling salesman problem, Formulation of Travelling Salesman problem as an Assignment problem, Solution procedure.
Mathematical formulation of Transportation problem, Tabular representation, Methods to find initial basic feasible solution, North West corner rule, Lowest cost entry method, Vogel's approximation methods, Optimality test, Method of finding optimal solution, Degeneracy in transportation problem, Method to resolve degeneracy, Unbalanced transportation problem.

Unit III

Concept of Dynamic programming, Bellman's principle of optimality, characteristics of Dynamic programming problem, Backward and Forward recursive approach, Minimum path problem, Single Additive constraint and Multiplicatively separable return, Single Additive constraint and Additively separable return, Single Multiplicatively constraint and Additively separable return.

Unit-IV

Historical development of CPM/PERT Techniques - Basic steps - Network diagram representation - Rules for drawing networks - Forward pass and Backward pass computations - Determination of floats - Determination of critical path - Project evaluation and review techniques updating.

Text Books:

[1] S. D. Sharma, Operations Research.
[2] Kanti Swarup, P. K. Gupta and Manmohan, Operations Research.
[3] H. A. Taha, Operations Research – An Introduction.

M.Sc.(Applied Mathematics)
Operation Research

Operation Research

AM 354(A) Paper IV(A) Semester III

Practical Questions

1. Find a geometrical interpretation and solution as well for the following LPP
Maximize $z=3x_1+ 5x_2$ subject to restrictions: $x_1+ 2x_2\leq 2000$, $x_1+ x_2\leq 1500$, $x_2\leq 600$ and $x_1\geq 0$, $x_2\geq 0$.
2. Solve the following LPP geometrically
Max. $z=8000\, x_1+7000x_2$, subject to $3x_1+ x_2\leq 66$, $x_1+ x_2\leq 45$, $x_1\leq 20$, $x_2\leq 40$ and $x_1\geq 0$, $x_2\geq 0$.
3. Using Simplex method, solve the following LPP
Min. $z= x_1-3x_2+2x_3$ subject to $3x_1-x_2+3x_3\leq 7$, $-2x_1+4x_2\leq 12$, $-4x_1+3x_2+8x_3\leq 10$, and x_1, x_2, $x_3\geq 0$.
4. Use two-phase simplex method to solve the problem
Min. $z= x_1-2x_2-3x_3$ subject to the constraints $-2x_1+x_2+3x_3=2$, $2x_1+3x_2+4x_3=1$, and x_1, x_2, $x_3\geq 0$.
5. Solve by using Big-M Method for the problem
Max. $z=-2x_1-x_2$, subject to $3x_1+x_2=3$, $4x_1+3x_2\geq 6$, $x_1+2x_2\leq 4$ and $x_1\geq 0$, $x_2\geq 0$.
6. A department head has four subordinates, and four tasks to be performed. Subordinates differ in efficiency and tasks in their intrinsic difficulty. Time each man would take to perform each task is given in effectiveness matrix. How the tasks should be allocated to each person so as to minimize the total man-hour.

		Subordinates			
		I	II	III	IV
Tasks	A	8	26	17	11
	B	13	28	4	26
	C	38	19	18	15
	D	19	26	24	10

7. There are 5 jobs to be assigned on 5 machines and associated cost matrix is as follows:

		Machines				
		I	II	III	IV	V
Jobs	A	11	17	8	16	20
	B	9	7	12	6	15
	C	13	16	15	12	16
	D	21	24	17	28	26
	E	14	10	12	11	15

Find the optimum assignment and associated cost using assignment technique.

8. Given the matrix of set-up costs, show how to sequence the production so as to minimize the set up cost per cycle.

	A_1	A_2	A_3	A_4	A_5
A_1	∞	2	5	7	1
A_2	6	∞	3	8	2
A_3	8	7	∞	4	7
A_4	12	4	6	∞	5
A_5	1	3	2	8	∞

9. Consider the following transport problem

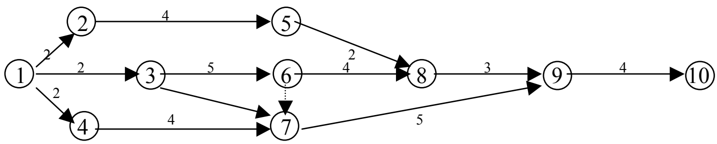
Source	Destination				Total
	D_1	D_2	D_3	D_4	
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
Total	20	40	30	10	100

Determine the initial feasible solution.

10. Find the initial basic feasible solution of the following transport problem by North West Corner Method.

		Warehouse				Factory Capacity
		W_1	W_2	W_3	W_4	
Factory	F_1	19	30	50	10	7
	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
Warehouse Requirement		5	8	7	14	34

11. Find the value of max $(y_1y_2y_3)$ subject to $y_1+y_2+y_3\geq 5$ and $y_1, y_2, y_3\geq 0$.
12. Minimize $z=y_1^2+y_2^2+y_3^2$ subject to $y_1+y_2+y_3\geq 15$ and $y_1, y_2, y_3\geq 0$.
13. Use dynamic programming to show that $-\sum_{i=1}^{n} p_i log p_i$, subject to $\sum_{i=1}^{n} p_i = 1$ is maximum, when $p_1=p_2=...=p_n=1/n$.
14. Use the principle of optimality to find the maximum value of $z=b_1x_1+b_2x_2+...+b_nx_n$ where $x_1+x_2+...+x_n=c$, and $x_1, x_2,..., x_n\geq 0$, $b_1, b_2,..., b_n>0$.
15. Solve the following problem using dynamic programming: Minimize $z= y_1^2+y_2^2+...+y_n^2$ subject to the constraints $y_1=y_2=...=y_n=b$ and $y_1, y_2,..., y_n\geq 0$.
16. Consider a project given by the following Network diagram. Numbers along various activities represent the normal time of completion of that activity D_{ij}



Find minimum time of completion of the project.

17. A project has the following time schedule (in months)

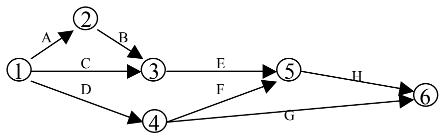
Activity	1→2	1→3	1→4	2→5	3→6	3→7	4→6	5→8	6→9	7→8	8→9
Normal Duration	2	2	1	4	8	5	3	1	5	4	3

Draw the network diagram to represent the above project. Find the critical path also.

18. A project has the following time schedule. Draw the Network diagram to represent this project. Find the critical path also.

Activity	1→2	1→3	1→4	2→5	3→6	3→7	4→7	5→8	6→8	7→9	8→9	9→10
Normal Duration	2	2	2	4	5	8	4	2	4	5	3	4

19. A project is represented by following Network diagram.



The time estimate of activities are as given below.

Activity	A	B	C	D	E	F	G	H
t_0	4	5	8	2	4	6	8	3
t_p	8	10	12	7	10	15	16	7
t_m	5	7	11	3	7	9	12	5

Determine the minimum expected time of completion of the project. Also determine the critical path.

20. A project has the following time schedule

Activity	Time in weeks	Activity	Time in weeks
1 →2	4	5 →7	8
1 →3	1	6 →8	1
2 →4	1	7 →8	2
3 →4	1	8 →9	1
3 →5	6	8 →10	8
4 →9	5	9 →10	7
5 →6	4		

Construct a PERT network and compute

(i) T_E and T_L for each event

(ii) Float for each activity and

(iii) critical path and its duration.

DEPARTMENT OF MATHEMATICS
OSMANIAUNIVERSITY

M.Sc. (Applied Mathematics)

AM – 304 B Semester III

Numerical Techniques

Paper IVB

Unit I

Transcendental and polynomial equations: Introduction, Bisection method, Iteration methods based on first degree equation; Secant method, Regulafalsi method, Newton-Raphson method, Iteration method based on second degree equation; Mullers method, Chebyshev method, Multipoint iterative method, Rate of convergence of secant method, Newton Raphson method,(Algorithms of above methods)

Unit II

System of linear algebraic equation: Direct methods, Guass elimination method, Triangularization method, Cholesky method, Partition method, Iteration method: Gauss seidel Iterative method, SOR method.

Unit III

Interpolation and Approximation: Introduction,Lagrange and Newton's divided difference interpolation,Finite difference operators, stirling and Bessel interpolation, Hermiteinterpolation,piecewise and Spline Interpolation,least square approximation.(Algorithms on Lagrange and Newton divided difference Interpolation).

Unit IV

Numerical Differentiation: methods based on Interpolation, methods based on Finite difference operators Numerical Integration: methods based on Interpolation, Newton's cotes methods, methods based on Undertermined coefficients, Guasslegendre Integration method, Numerical methods ODE: Singlestep methods: Eulers method, Taylor series method, Rungekutte second and forth order methods, Multistep methods: Adam Bash forth method, Adam Moulton methods, Milne-Simpson method. (Algorithms on Trapezoidal, Simpson, Eulers&Runggekutte. methods only)

Text Book:

[1] Numerical Methods for Scientific and Engineering computation by M.K. Jain, S.R.K. Iyengar, R.K. Jain, New Age Int. Ltd., New Delhi.

[2] Computer Oriented Numerical Methods by V. Rajaraman.

Reference:

[1] Introduction to Numerical Analysis, by S.S. SastryPrentice Hall Flied.

M.Sc. (Applied Mathematics)

AM354B Numerical Techniques **Semester –III**

Practical Questions

Paper –IVB

1. Use the bisection method to find the solution accurate to within 10^{-5} for the problem $2x\cos(2x)-(x+1)^2=0 \ for \ -3\le x\le 2$
2. Use the newton-Raphson method to find the solution accurate to within 10^{-5} for the problem $\sin x-e^x=0 \ for \ 0\le x\le 1, 3\le x\le 4, 6\le x\le 7$
3. The fourth degree polynomial $f(x)=230x^4+18x^3+9x^2-22x$ has two real zeros, one in [-1,0] and the other in [0,1]. Attempt the approximate these zeros to within 10^{-6}
   (a) Method of False position (b) Secant method (c)Mullers Method (d)chebeshev method (e)Multipoint iterative method
4. Use LU decomposition method to solve the system of equations $2x_1-x_2=1, \ -x_1+2x_2-x_3=0, \ -x_2+2x_3-x_4=0, \ -x_3+2x_4=1$
5. Use Cholesky method to find the factorization of the matrix $A=LL'$ for the matrix $\begin{bmatrix} 4 & -1 & 1 \\ -1 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$
6. Explain Gauss- seidelinterative method to solve the system of equations Ax=b
7. Use Gauss-Seidel method to solve the system of equations $x_1+2x_2-2x_3=-1, \ x_1+x_2+x_3=2, \ 2x_1+2x_2+x_3=5,$
8. The linear system Ax=b given by $4x_1+3x_2=24, \ 3x_1+4x_2-x_3=30, \ -x_2+4x_3=-24,$ has the solution $(3,4,-5)^T$ compare the iteration from the Gauss-Seidel method and the SOR method with w=1.25 using $x^{(0)}=(1,1,1)^T$ for the methods.
9. Use appropriate lagrange interpolating polynomials degree one, two and three to approximate $f(0.43)$ if $f(0.1)=1, f(0.25)=1.64872, f(0.5)=2.71828, f(0.75)=4048169$
10. Derive Newton's divided difference interpolating polynomial.
11. Compute the divided difference polynomial for the data

x_i	1.0	1.3	1.6	1.9	2.2
$f(x_i)$	0.7651977	0.6200860	0.4554022	0.2818156	0.1103623

12. Derive Hermiteinterpolateory polynomial.
13. Use the Hermite polynomial that agrees with the data listed in table to find approximation of $f(1.5)$,

x_i	$f(x_i)$	$f'(x_i)$
1.3	0.6200860	-0.5220232
1.6	0.4554022	-0.5698959
1.9	0.2818186	-0.5811571

14. Apply Taylor's method of order two and four with N=10 to the initial value problem
    $y' = y - t^2 + 1,\ 0 \le t \le 2,\ y(0) = 0.5$
15. Derive Runge-Kutta method of order Two.
16. Use the Runge-Kutta method of order 4 with h=0.2, N=10 and $t_i = 0.2$; to obtain approximation to the solution of the initial value problem
    $y' = y - t^2 + 1,\ 0 \le t \le 2,\ y(0) = 0.5$
17. Derive Adams-Moulton method of order 4.
18. Derive Adams-Bashforth method of order 4.

Department of Mathematics,
Osmania University
M.Sc. Applied Mathematics

AM 304(C) **Dynamical Systems** **Semester III**

Paper IV (C)

Unit I :

Linear Oscillators and Predictability, Damped and Driven Nonlinear Oscillators, Nonlinear Oscillations and Bifurcations.

Unit II :

Autonomous and Nonautonomous Systems, Dynamical Systems and Coupled First Order Differential Equations, Phase Space/ Phase Plane and Phase Trajectories, Classification of Equilibrium Points, Limit Cycle Motion-Periodic Attractor, Higher dimensional Systems, More Complicated Attractors.

Unit III :

Some Simple Bifurcations, Discrete Dynamical Systems, Strange Attractor in Henon Map.

Unit IV :

Bifurcation Scenario in Duffing Oscillator, Lorenz Equations, Solving the Problems of Dynamical Systems using an online software.

Text Book : Nonlinear Dynamics : Inerrability, Chaos, and Patterns.
M.Lakshmanan, S.Rajasekhar, 2003, Springer.

Reference Books :

(1) Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields. **J.Guckenheimer, P.Holmes**, 1983, Springer.

(2) Nonlinear Dynamics and Chaos: With Applications To Physics, Biology, Chemistry, and Engineering (Studies in Nonlinearity), **Steven H. Strogatz**, 2001, Addison-Wesley.

M.Sc. Applied Mathematics

Dynamical Systems

AM 354(C) **Paper IV (C)** **Semester III**

Practical Questions

(1) Obtained the solution of the undamped, forced linear harmonic oscillator

$$\ddot{x} + \omega_0^2 x = f sin(\omega t)$$

and indentify the nature of resonance.

(2) Analyze the dynamics of the damped linear harmonic oscillator driven by the combined periodic external forcing of different frequencies,

$$\ddot{x} + \alpha\dot{x} + \omega_0^2 x = f_1 \sin \omega t, \quad \alpha > 0$$

(3) Obtain the systems instead of being driven by external forces can also be exited by parametric modulation. Consider the sinusoidal parametric excitation of the linear harmonic oscillator described by the Mathieu equation

$$\ddot{x} + \omega_0^2(1 + \varepsilon \cos \omega t)x = 0, \quad \varepsilon \ll 1$$

Discuss the dynamics underlying the system

(4) Obtain the frequency-response relations and draw the primary resonance curves for the following nonlinear oscillators. Also discuss the nature of the secondary resonances. Compare the dynamics between the various systems.

(a) Quadratic Duffing oscillator

$$\ddot{x} + \alpha\dot{x} + \omega_0^2 x + \beta x^2 = f \sin \omega t$$

(b) Periodically driven elastic oscillator

$$\ddot{x} + \alpha\dot{x} + \omega_0^2 x + \beta x^2 + \gamma x^3 = f \sin \omega t$$

(5) Consider the following parametrically modulated nonlinear oscillators and investigate the underlying *parametric resonances* and frequency response relations.

(a) Parametrically driven Duffing oscillator :

$\ddot{x} + \alpha\dot{x} + \omega_0^2(1 + \eta \sin \omega t)x + \beta x^3 = 0$, where η is a parameter.

(b) Parametrically driven Van der Pol oscillator :

$$\ddot{x} - \alpha(1 - x^2)\dot{x} + \omega_0^2(1 + \eta \sin \omega t)x = 0$$

(6) The equation of motion of the damped cubic anharmonic oscillator is $\ddot{x} + d\dot{x} - ax + bx^3 = 0$, wher *d,a* and *b* are positive constants. Find the equilibrium points and investigate their stability.

(7) Applying Bendixson's nonexistence theorem, discuss the possibility of occurrence of a limit cycle in the following systems. Also investigate analytically/numerically the nature of solutions in these systems.

(a) Bonhoeffer-van der Pol oscillator

$\dot{x} = x - \frac{x^3}{3} - y + A_0,\ \ \dot{y} = c(x + a - by)$, (A_0, a, b, c : constants)

(b) Brusselator equations

$$\dot{x} = a - x - bx + x^2 y,\ \ \dot{y} = bx - x^2 y$$

(8) Find the equilibrium points and analyze their linear stability of Lorenz equations.

(9) Show that the Routh-Hurwitz's necessary conditions for an equilibrium point of the Rössler equations

$\dot{x} = -(y + z), \dot{y} = x + ay, \dot{z} = b + z(x - c)$ to be stable are

$$a_1 = c - a - x^* > 0\ , a_3 = c - x^* - az^* > 0,$$

$$a_1 a_2 - a_3 = (c - x^*)z^* - a(c - x^*)^2 + a^2(c - x^*) - a > 0$$

(10) Numerically integrating the following dynamical system, obtain the quasiperiodic and chaotic solutions for the specified parametric values. Draw the phase portrait and Poincaré map of these attracting solutions

(a) Brusselator equation:

$$\dot{x} = A + x^2 y - Bx - x + f\cos\omega t\ ,\qquad \dot{y} = Bx - x^2 y$$

For A = 0.4 and B = 1.2, quasiperiodic motion occurs for f = 0.005 and w = 1.5, while chaotic dynamics occurs for f = 0.08 and w =0.86.

(11) Study the occurrence of saddle-node bifurcation in the system

$$\dot{x} = \mu + x^2\ ,\ \ \dot{y} = -y$$

using both linear stability analysis and exact solutions.

(12) Verify that the following systems undergo pitchfork bifurcation at $\mu = 0$. Sketch the bifurcation diagrams and the phase portraits of the corresponding systems.

$$(\boldsymbol{a})\quad \dot{\boldsymbol{x}} = -\boldsymbol{\mu x} - \boldsymbol{x}^3,\quad \dot{\boldsymbol{y}} = -\boldsymbol{y}$$
$$(\boldsymbol{b})\quad \dot{\boldsymbol{x}} = -\boldsymbol{\mu x} + \boldsymbol{x}^3,\quad \dot{\boldsymbol{y}} = -\boldsymbol{y}$$

(13) Discuss the occurrence of transcritical bifurcation in the system

$$\dot{\boldsymbol{x}} = -\boldsymbol{x}(\boldsymbol{x}^2 - 2\boldsymbol{bx} - \boldsymbol{a}),\quad \dot{\boldsymbol{y}} = -\boldsymbol{y}$$

(14) Find the conditions for which the equilibrium point (0,0) of the system

$$\dot{\boldsymbol{x}} = \boldsymbol{y} - \boldsymbol{x}(\boldsymbol{x}^2 + \boldsymbol{y}^2 - \boldsymbol{\alpha}),\ \dot{\boldsymbol{y}} = -\boldsymbol{x} - \boldsymbol{y}(\boldsymbol{x}^2 + \boldsymbol{y}^2 - \boldsymbol{\alpha})$$

where $\boldsymbol{\alpha}$ is a constant, undergoes a Hopf bifurcation.

(15) Verify that the criterion for stability of period-4 solution is

$$|\boldsymbol{f}'(\boldsymbol{x}_1^*)\boldsymbol{f}'(\boldsymbol{x}_2^*)\boldsymbol{f}'(\boldsymbol{x}_3^*)\boldsymbol{f}'(\boldsymbol{x}_4^*)| < \mathbf{1}$$

(16) Show that for the linear map $x_{n+1} = ax_n$, the Lyapunov exponent is negative for bounded solution and so there is no sensitive dependence on initial conditions.

(17) Investigate the bifurcations phenomena in the Duffing oscillator equation

$$\ddot{x} + \alpha\dot{x} + \omega_0^2 x + \beta x^3 = f\sin\omega t\,,\quad \alpha > 0$$

for (i) the single-well, $\omega_0^2 > 0,\ \ \beta > 0$, and (ii) double-hump, $\omega_0^2 > 0,\ \ \beta < 0,$ potentials.

(18) Analyze the various bifurcations and routes to chaos in the driven Vander Pol oscillator.

(19) Verify that for the invertible and monotonically increasing map $X_{n+1} = A\, exp(x_n)$, $A > 0$ the only possible attractor is period-1 equilibrium point.

(20) The Brusselator model equation describing hypothetical three-molecular chemical reaction with autocatalytic step under far-from-equilibrium conditions is

$$\dot{x} = A + x^2y - Bx - x + f cos\ \omega t\,,\ \ \dot{y} = Bx - x^2y$$

For $A = 0.4$ and $B = 1.2$, investigate the occurrence of periodic, quasiperiodic and chaotic motions in $f - \omega$ parameter space with $f\epsilon[0,0.4]$ and $\omega\epsilon[0,4]$.

Departemnt of MATHEMATICS,OU
Proposed Choice Based Credit System (CBCS)
M.Sc Applied Mathematics

Semester -IV

S.No.	Code No	Paper	Paper Title	Hrs/Week	Internal Assessment	Semester Exam	Total Marks	Credits
1. Core	AM 401	I	Advanced Complex Analysis	4	20	80	100	4
2. Core	AM 402	II	Finite Element Methods	4	20	80	100	4
3. Elective	AM 403(A) AM 403(B) AM 403(C)	III (A) III (B) III (C)	Integral Equations & Calculus of variations MHD Bio-Mechanics	4	20	80	100	4
4.Elective	AM 404(A) AM 404(B) AM 404 (C) AM 404 (D)	IV(A) IV(B) IV(C) IV(D)	Functional Analysis Discrete Mathematics Topology Project	4 OR 6	20 ….	80 ….	100 150	4 OR 6
5. Practicals	AM 451	Practical	Advanced Complex Analysis	4	….	50	50	2
6. Practicals	AM 452	Practical	Finite Element Methods	4	….	50	50	2
7. Practicals	AM 453 (A) AM 453 (B) AM 453 (C)	Practical	Integral Equations & Calculus of variations MHD Bio - Mechanics	4	….	50	50	2
8. Practicals	AM 454 (A) AM 454 (B) AM 454 (C)	Practical	Functional Analysis Discrete Mathematics Topology	4	….	50	50	2
			Total :	**32**	**80**	**520**	**600**	**24**
9.Seminar			Seminar	2	….	….	25	1

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. Applied Mathematics
AM401 Semester IV

Advanced Complex Analysis

Paper-I

UNIT-I

Entire Functions: Jensen's formula -Functions of finite order- Infinite products Generalities - Example: the product formula for the sine function -Weierstrass infinite products - Hadamard's factorization theorem

UNIT-II

The Gamma and Zeta Functions: The gamma function –Analytic continuation-Further properties of Γ -The zeta function -Functional equation and analytic continuation

UNIT-III

The Zeta Function and Prime Number Theorem: Zeros of the zeta function - Estimates for $1/\zeta(s)$ - Reduction to the functions ψ and ψ_1 -Proof of the asymptotics for ψ_1 - Note on interchanging double sums

UNIT-IV

Conformal Mappings: Conformal equivalence and examples -The disc and upper half-plane - Further examples -The Dirichlet problem in a strip -The Schwarz lemma; automorphisms of the disc and upper half-plane-Automorphisms of the disc - Automorphisms of the upper half-plane

Text Book : Elias M Stein, Rami Shakarchi , C*omplex Analysis*

References: Lars V Ahlfors, *Complex Analysis*

R P Boas, *Entire Functions*

Lars V Ahlfors, *Conformal Invariants*

Advanced Complex Analysis

Paper-I

AM451 Practical Questions Semester-IV

1

Prove that if $|z| < 1$, then

$$(1+z)(1+z^2)(1+z^4)(1+z^8)\cdots = \prod_{k=0}^{\infty}(1+z^{2^k}) = \frac{1}{1-z}.$$

2

Find the Hadamard products for:

(a) $e^z - 1$;

(b) $\cos \pi z$.

3

Prove that for every z the product below converges, and

$$\cos(z/2)\cos(z/4)\cos(z/8)\cdots = \prod_{k=1}^{\infty}\cos(z/2^k) = \frac{\sin z}{z}.$$

4

Show that the equation $e^z - z = 0$ has infinitely many solutions in $\mathbb{C}$.

5

Prove Wallis's product formula

$$\frac{\pi}{2} = \frac{2\cdot 2}{1\cdot 3}\cdot\frac{4\cdot 4}{3\cdot 5}\cdots\frac{2m\cdot 2m}{(2m-1)\cdot(2m+1)}\cdots.$$

6

Prove that

$$\Gamma(s) = \lim_{n\to\infty}\frac{n^s n!}{s(s+1)\cdots(s+n)}$$

whenever $s \neq 0, -1, -2, \ldots$.

7

Show that Wallis's product formula can be written as

$$\sqrt{\frac{\pi}{2}} = \lim_{n\to\infty}\frac{2^{2n}(n!)^2}{(2n+1)!}(2n+1)^{1/2}.$$

8

Use the fact that $\Gamma(s)\Gamma(1-s) = \pi/\sin \pi s$ to prove that

$$|\Gamma(1/2+it)| = \sqrt{\frac{2\pi}{e^{\pi t}+e^{-\pi t}}}, \qquad \text{whenever } t\in\mathbb{R}.$$

9

The **Beta function** is defined for $\mathrm{Re}(\alpha) > 0$ and $\mathrm{Re}(\beta) > 0$ by

$$B(\alpha,\beta) = \int_0^1 (1-t)^{\alpha-1}t^{\beta-1}\,dt.$$

(a) Prove that $B(\alpha,\beta) = \dfrac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$.

(b) Show that $B(\alpha,\beta) = \displaystyle\int_0^\infty \frac{u^{\alpha-1}}{(1+u)^{\alpha+\beta}}\,du$.

10

Prove that for $\mathrm{Re}(s) > 1$,

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{e^x - 1}\, dx.$$

11

Prove as a consequence that one has

$$(\zeta(s))^2 = \sum_{n=1}^\infty \frac{d(n)}{n^s} \quad \text{and} \quad \zeta(s)\zeta(s-a) = \sum_{n=1}^\infty \frac{\sigma_a(n)}{n^s}$$

for $\mathrm{Re}(s) > 1$ and $\mathrm{Re}(s-a) > 1$, respectively. Here $d(n)$ equals the number of divisors of n, and $\sigma_a(n)$ is the sum of the a^{th} powers of divisors of n. In particular, one has $\sigma_0(n) = d(n)$.

12

Show that if $\{a_m\}$ and $\{b_k\}$ are two bounded sequences of complex numbers, then

$$\left(\sum_{m=1}^\infty \frac{a_m}{m^s}\right)\left(\sum_{k=1}^\infty \frac{b_k}{k^s}\right) = \sum_{n=1}^\infty \frac{c_n}{n^s} \quad \text{where } c_n = \sum_{mk=n} a_m b_k.$$

The above series converge absolutely when $\mathrm{Re}(s) > 1$.

13

Prove that for $\mathrm{Re}(s) > 1$

$$\frac{1}{\zeta(s)} = \sum_{n=1}^\infty \frac{\mu(n)}{n^s},$$

where $\mu(n)$ is the **Möbius function** defined by

$$\mu(n) = \begin{cases} 1 & \text{if } n = 1\,, \\ (-1)^k & \text{if } n = p_1 \cdots p_k, \text{ and the } p_j \text{ are distinct primes}\,, \\ 0 & \text{otherwise}\,. \end{cases}$$

Note that $\mu(nm) = \mu(n)\mu(m)$ whenever n and m are relatively prime. [Hint: Use the Euler product formula for $\zeta(s)$.]

14

Show that

$$\sum_{k|n} \mu(k) = \begin{cases} 1 & \text{if } n = 1, \\ 0 & \text{otherwise.} \end{cases}$$

15

Prove that the Dirichlet series

$$\sum_{n=1}^\infty \frac{a_n}{n^s}$$

converges for $\mathrm{Re}(s) > 0$ and defines a holomorphic function in this half-plane.

16

Does there exist a holomorphic surjection from the unit disc to $\mathbb{C}$?

17

Prove that $f(z) = -\frac{1}{2}(z + 1/z)$ is a conformal map from the half-disc $\{z = x + iy : |z| < 1,\ y > 0\}$ to the upper half-plane.

18

Prove that the function u defined by

$$u(x,y) = \mathrm{Re}\left(\frac{i+z}{i-z}\right) \qquad \text{and} \qquad u(0,1) = 0$$

is harmonic in the unit disc and vanishes on its boundary. Note that u is not bounded in $\mathbb{D}$.

19

Show that if $f : D(0,R) \to \mathbb{C}$ is holomorphic, with $|f(z)| \leq M$ for some $M > 0$, then

$$\left| \frac{f(z) - f(0)}{M^2 - \overline{f(0)}f(z)} \right| \leq \frac{|z|}{MR}.$$

20

Prove that if $f : \mathbb{D} \to \mathbb{D}$ is analytic and has two distinct fixed points, then f is the identity, that is, $f(z) = z$ for all $z \in \mathbb{D}$.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. Applied Mathematics
Paper II

AM – 402 Semester IV

Finite Element Methods

Unit- I

Weighted residual methods: - Least square method - Partition method - Galerkin method - Moment method - Collocation method – Variational Methods: Ritz method. (one- dimensional only)

Unit -II

Finite Elements - Line segment elements - Triangular element - Rectangular elements with examples.

Unit- III

Finite Element Methods: Ritz finite element method - Least square finite element method - Galerkin finite element method - Boundary value problem in ordinam differential equations- Assembly of element equations- Boundary value problem in PDE-Linear tringular element- Mixed boundary conditions- Boundary points- Examples.

Unit-IV

Eigen value problems- Finite Element Error Analysis- Approximation Errors- Various measures of Errors- Convergence of solution- Accuracy of the solution- Examples. (5.1 to 5.4) of [2]

Text Books:

[1] M.K.Jain, Numerical Solution of Differential Equations.
New Age Int.(P).Ltd., New Delhi.(for Units I, II and III)
[2] J. N. Reddy, Finite Element Methods, McGraw-Hill International Edition, Engineering Mechanics Series. (for Unit IV)

M.Sc. (Applied Mathematics)
Finite Element Methods

AM 452	Paper II	Semester IV

Practical Questions

1. Consider the boundary value problem $u''+(1+x^2)u+1=0$, $u(1)=u(-1)=0$. Determine the coefficients of the approximate solution $w(x)=a_1(1-x^2)+a_2x^2(1-x^2)$ by using partition method.
2. Consider the boundary value problem $u''+(1+x^2)u+1=0$, $u(1)=u(-1)=0$. Determine the coefficients of the approximate solution $w(x)=a_1(1-x^2)+a_2x^2(1-x^2)$ by using least square method.
3. Solve the BVP $-u_{xx}-u+x^2=0$ for $0<x<1$, $u(0)=0$, $u'(1)=1$.
4. Consider the boundary value problem $u''+(1+x^2)u+1=0$, $u(1)=u(-1)=0$. Determine the coefficients of the approximate solution $w(x)=a_1(1-x^2)+a_2x^2(1-x^2)$ by using Galerkin method.
5. Consider the IVP $u_t=u_{xx}$, $u(x,0)=\cos(\pi x/2)$, $-1\leq x\leq 1$, $u(-1,t)=u(1,t)=0$. Use the Galerkin method with approximate solution of the form $w(x,t)=(1-x^2)(1-4x^2)u_0(t)+(16/3)(x^2-x^4)u_1(t)$ where $u_0(t)$ and $u_1(t)$ are unknown solution values at the nodes 0 and ½ respectively.
6. For the element $e_i=[-1,1]$ obtain the Hermite cubic polynomial.
7. For the line element $e_i=[-1,1]$ obtain the shape function and the local coordinates. Also obtain the linear Lagrange polynomial for this element.
8. Obtain the linear quadratic Lagrange polynomial for the triangular element with the nodes 1(0,0), 2(2,0), 3(1,1).
9. Obtain the linear quadratic Lagrange polynomial for the triangular element with the nodes 1(0,0), 2(5,0), 3(0,5).
10. Obtain the linear quadratic Lagrange polynomial for the line element $e_i=[-1,1]$.
11. Solve by the finite element method (Ritz method) the BVP $u''+(1+x^2)u+1=0$, $u(1)=u(-1)=0$.
12. Solve the differential equation $u''-u=1$ by Galerkin Method with $u(1)=u(-1)=0$.
13. Consider the BVP
$$u_{xx}+u_{yy}=-1, \quad |x|\leq 1,\ |y|\leq 1$$
$$u=0, \quad |x|=1,\ |y|=1$$
Use the Galerkin method to determine the solution values at the nodes (0,0), (1/2,0) and (1/2,1/2).
14. Use the finite element method (Ritz method) to solve the BVP
$$u_{xx}+u_{yy}=-1, \quad |x|\leq 1,\ |y|\leq 1$$
$$u=0, \quad |x|=1,\ |y|=1$$
with $h=1/2$.
15. Solve by the least square finite element method the BVP $u''+(1+x^2)u+1=0$, $u(1)=u(-1)=0$.
16. Verify the error estimates in $||u-u_h||_0\leq c_1h^2$, $||u'-u_h'||\leq c_2h$, for the differential equation $-u_{xx}=2$ for $0<x<1$ with $u(0)=u(1)=0$.
17. Consider the differential equation $-u_{xx}=\lambda u$, $0<x<1$, $u(0)=u(1)=0$. Determine the value(s) of λ and u for which the differential equations and the boundary conditions are satisfied.

18. Solve the first order Hyperbolic equation $u_t+cu_x=0$ with appropriate initial and boundary conditions where c is constant.
19. Find the solution of IBVP $u_{tt}=u_{xx}$, $0\leq x\leq 1$ subject to $u(x,0)=\sin\pi x$, $u_t(x,0)=0$, $0\leq x\leq 1$ and $u(0,t)=u(1,t)=0$.
20. Formulate eigen-value problem for the Parabolic PDE

$$\rho cA\frac{\partial u}{\partial t}-\frac{\partial}{\partial x}\left(KA\frac{\partial u}{\partial x}\right)=q(x,t).$$

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Applied Mathematics)

AM – 403A Paper IIIA Semester IV

Integral Equations & Calculus of Variations

Integral Equations:

Unit I

Volterra Integral Equations: Basic concepts - Relationship between Linear differential equations and Volterra Integral equations - Resolvent Kernel of Volterra Integral equation. Differentiation of some resolvent kernels - Solution of Integral equation by Resolvent Kernel - The method of successive approximations - Convolution type equations - Solution of Integro-differential equations with the aid of the Laplace Transformation – Volterra integral equation of the first kind - Euler integrals - Abel's problem - Abel's integral equation and its generalizations.

Unit II

Fredholm Integral Equations: Fredholm integral equations of the second kind – Fundamentals – The Method of Fredholm Determinants - Iterated Kernels constructing the Resolvent Kernel with the aid of Iterated Kernels - Integral equations with Degenerated Kernels. Hammerstein type equation - Characteristic numbers and Eigen functions and its properties.

Green's function: Construction of Green's function for ordinary differential equations - Special case of Green's function - Using Green's function in the solution of boundary value problem.

Calculus of Variations:

Unit III

The Method of Variations in Problems with fixed Boundaries:

Definitions of Functionals – Variation and Its properties - Euler's' equation - Fundamental Lemma of Calculus of Variation-The problem of minimum surface of revolution - Minimum Energy Problem Brachistochrone Problem - Variational problems involving Several functions - Functional dependent on higher order derivatives - Euler Poisson equation.

Unit IV

Functional dependent on the Functions of several independent variables - Euler's equations in two dependent variables - Variational problems in parametric form - Application of Calculus of Variation - Hamilton's principle - Lagrange's Equation, Hamilton’s equations.

Text Books:

[1] M. KRASNOV, A. KISELEV, G. MAKARENKO, Problems and Exercises in Integral Equations (1971)

[2] S. Swarup, Integral Equations, (2008)

[3] L.ELSGOLTS, Differential Equation and Calculus of Variations, MIR Publishers, MOSCOW

M.Sc.(Applied Mathematics)

Integral Equations & Calculus of Variations

AM453A | Paper IIIA | Semester IV

Practical Questions

1. From an Integral equation corresponding to the differential equation $y''' + xy'' + (x^2 - x)y = xe^x + 1$; $y(0) = y'(0) = 1;\ y''(0) = 1$
2. Convert the differential equation $y''' + xy'' + (x^2 - x)y = xe^x + 1$; with initial conditions $y(0) = y'(0) = 1,\ y''(0) = 0$; into Volterra's Integral Equations.
3. Solve the Integral Equations $\varphi''(x) + \varphi(x) + \int Sinh(x-t)\varphi(t)dt + \int_0^x Cosh(x-t)\varphi'(t)dt = Coshx$; $\varphi(0) = \varphi'(0) = 0$.
4. Solve the Integral Equations $\int_0^x \frac{\varphi(t)dt}{(x-t)^\alpha} = x^n$; $0 < \alpha < 1$;
5. With the aid of Resolvent Kernel, find the solution of the Integral equation $$\varphi(x) = e^x \sin x + \int_0^x \frac{2+\cos x}{2+\cos t}\varphi(t)dt$$
6. Solve the Integral Equations $\phi(x) - \lambda\int_0^1 \arccos t.\phi(t)dt = \frac{1}{\sqrt{1-x^2}}$
7. Find the Characteristic numbers and Eigen function of the Integral Equations $$\phi(x) - \lambda\int_0^1 \left(45x^2 \log t - 9t^2 \log x\right)\phi(t)dt = 0$$
8. **Applications of Green's function :** Construct Green's function for the homogeneous boundary value problem $y^{iv}(x) = 0;\ y(0) = y'(0) = 0;\ y(1) = y'(1) = 0$.
9. **Applications of Green's function :** Solve the Boundary Value problem $y^{iv}(x) = 1;\ y(0) = y'(0) = y''(1) = y'''(0) = 0$.
10. **Applications of Green's function :** Solve the Boundary Value problem $y'' + y = x^2$; $y(0) = y(\pi/2) = 0$.
11. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{1+y'^2}}{y}dx$
12. Test for an extremum the functional $v[y(x)] = \int_0^1 \left(xy + y^2 - 2y^2 y'\right)dx$; $y(0) = 1;\ y(1)) = 2$.

13. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} (16y^2 - y''^2 + x^2)dx$

14. Determine the extremals of the functional $v[y(x)] = \int_{-l}^{l} (\frac{\mu}{2} y''^2 + \rho y)dx$ that satisfies the boundary conditions $y(-l) = y'(-l) = y(l) = y'(l) = 0$

15. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} (2yz - 2y^2 + y'^2 - z'^2)dx$

16. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} \left[y^2 + (y')^2 + \frac{2y}{Coshx} \right] dx$

17. Write the ***Ostrgradsky*** equation for the functional $v[z(x,y)] = \iint_D \left[\left(\frac{\partial z}{\partial x} \right)^2 - \left(\frac{\partial z}{\partial y} \right)^2 \right] dxdy$

18. **Applications of Hamilton's and Lagrange's equations:** Derive the equation of a vibrations of a Rectilinear Bar.
19. **Applications of Hamilton's and Lagrange's equations:** A particle of mass **m** is moving vertically under the action of gravity and a resistance force numerically equal to **k** times the displacement **x** from an equilibrium position. Obtain the Hamilton's and Euler's equation.
20. Use Hamilton's principle to find the equations for the small vibrations of a flexible stretching string of length ***l*** and tension **T** fixed at end points.

Department of Mathematics
Osmania University
M.Sc. Applied Mathematics
Magnetohydrodynamics

AM 403 B **Paper III (B)** **Semester IV**

Syllabus

Unit I:

Governing equations of electrohydrodynamics: The electric field and Lorentz force, Ohm's law and volumetric Lorentz force, Ampere's law, Faraday's law in differential form, reduced form of Maxwell equations for MHD, transport equation for imposed magnetic field (**B**), an important kinematic equation, the significance of Faraday's law and Faraday's law in ideal conductors.

Unit II:

Vorticity, angular momentum and Biot-Savart Law; Advection and diffusion of vorticity, Kelvin's theorem, Helmholtz law and helicity, Prandtl-Batchelor theorem; Fluid flow in the presence of Lorentz force: Equations of MHD and dimensionless groups. Maxwell stresses.

Unit III:

Kinematics of MHD: Analogy to vorticity, Diffusion of a magnetic field, Advection in ideal conductors: Alfven's theorem, Manetic helicity.

Unit IV:

Advection and diffusion. Azimuthal field generation by differential rotation. Magnetic reconnection.

Textbook:

P.A. Dadvidson, "An Introduction to magnetohydrodynamics", Cambridge University Press, 2001.

Reference books:

1. PH Roberts, An Introduction to magnetodynamics", Longmans' Publishers, 1961.
2. HK Moffat, Magnetic field generation in electrically conducting fluids, Cambridge Universiyt Press, 1978.
3. R Moreau, Magneto hydrodyanimcs, Kluwer Academy Publishers, 1990.

Magnetohydrodynamics
Practical Questions

Code No. AM 453 (B) **Paper III (B)** **Semester IV**

1. Show that, for a force-free field, $(\nabla^2 + \alpha^2)\mathbf{B} = 0$, where α is a constant and **B** is an imposed magnetic field.
2. Faraday's law implies that $\frac{\partial}{\partial t}(\nabla.\mathbf{B}) = 0$. If this is also true relative to all set axis moving uniformly relative to one another then show that $\nabla.\mathbf{B} = 0$.
3. Write the reduced form of Maxwells equations for MHD.
4. Use Faraday's law and Ampere's law show that $\frac{d}{dt}\int_V (\boldsymbol{B^2}/2\mu)dV = -\int_V \boldsymbol{J}.\boldsymbol{E}\, dV - \oint_S \left[\frac{\boldsymbol{E}\times\boldsymbol{B}}{\mu}\right].dS.$
5. Show that if, at time t = 0, there exists a force free field, $\nabla\times\boldsymbol{B} = \alpha\boldsymbol{B}$, in a stationary fluid, then that field will decay as $\boldsymbol{B} \sim \exp(-\lambda\alpha^2 t)$, remaining as a force-free field.
6. Suppose that $\boldsymbol{G}$ is a solenoidal field, $\nabla.\boldsymbol{G} = \boldsymbol{0}$, S_m is a surface which is embedded in a conducting medium. Then show that $\frac{d}{dt}\int_{S_m} \boldsymbol{G}.d\boldsymbol{S} = \int_{S_m}\left[\frac{\partial \boldsymbol{G}}{\partial t} - \nabla\times(\boldsymbol{u}\times\boldsymbol{G})\right].d\boldsymbol{S}$
7. Write the significance of Faraday's law.
8. A conducting fluid flows in a uniform magnetic field which is negligibly perturbed by the induced currents. Show that the condition for there to be no net change distribution in the field that $\boldsymbol{B}.(\boldsymbol{\nabla}\times\boldsymbol{u}) = 0$.
9. Determine an expression for vorticity ω in circular Poiseulle flow for which the velocity **V** = $V_z i_z$ where $V_z = V_m\left(1 - \frac{r^2}{a^2}\right)$ V_m being the maximum velocity and a is the tube radius. What is the shape of a vortex line.
10. If the velocity field $\boldsymbol{V} = 2xi + yj + z^2k$, then find the vorticity and helicity, when the fluid flow through the parallopiped x =0,1; y =0,1 and z =0,1.
11. If the fluid is incompressible and flow field is steady, then show that helicity is conserved.
12. Consider the energy equation $\frac{DT}{Dt} = \alpha\,\nabla^2 T$. Show that for laminar, high Peclect number, closed streamline flows, the temperature outside the boundary layer is constant.
13. Inviscid fluid occupies the region x ≥ 0 , Y > 0 bounded by two rigid boundaries x = 0, y =0. Its motion results wholly from the presence of a line vortex, which itself moves according to the Helmholtz vortex theorem. Show that the path taken by the vortex is $x^{-2}+y^{-2}$ = Constant.
14. Derive the Navier-Stoke's equation with Lorentz force.
15. Derive the magnetic induction equation using Maxwell's equations.
16. Consider a long conducting cylinder which, at *t = 0*, contains a uniform axial magnetic field, B_0. The field outside the cylinder is zero. The axial field inside the cylinder will decay according to the diffusion equation $\frac{\partial B}{\partial t} = \lambda\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial B}{\partial r})$ subject to *B = 0* at *r = R* and $B = B_0$ at *t = 0.* Show that a Fourier Bessel series of the form $B = \sum_{n=1}^{\infty} A_n J_0\left(\frac{\gamma_n r}{R}\right)\exp\left(-\frac{\gamma_n^2\lambda t}{R^2}\right)$ is a possible solution.
17. State and prove Alfven's theorem.
18. If the imposed magnetic field **B** is solenoidal and the fluid incompressible then prove that magnetic helicity is conserved for the steady flow.
19. Consider a magnetic filed **B** = $B_0\hat{e}_z$prevades a conducting fluid, and region of theis fluid, r< R, is in a state of rigid body rotation. In the steady state find the magnetic flux function with suitable conditions.
20. Write the role played by magnetic reconnection in the solar MHD.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. Applied Mathematics

Bio-Mechanics

AM 403C **Paper-IIIC** **SEMESTER-IV**

Unit I

Introduction - Continuum Approach -**Blood Flow in Heart, Lung, Arteries and Veins**: Introduction - The geometry of the circulation system - Field equations and Boundary conditions - Coupling of Left Ventricle to Aorta and Right Ventricle to Pulmonary Artery - Pulsatile Flow in Arteries - Progressive waves superposed on a Steady flow - Reflection and Transmission of Waves at Junctions - Velocity profile of a steady flow in a Tube - Steady Laminar Flow in Elastic Tube. Velocity Profile of Pulsatile flow. (1.1, 1.7, 5.1, 5.2, 5.4, 5.6 – 5.12 of [1]).

Unit II

The Reynolds Number, Stokes Number, and Womersley Number - Equations of Balance of Energy and Work - Systemic Blood Pressure - Flow in a Collapsible Tubes - **Micro and Macro Circulation**: Introduction - Major Feature of Microcirculation - The Rheological Properties of Blood - Pulmonary Blood Flow - Waterfall Phenomenon in Zone 2 - (5.13-5.17, 6.1, 6.3, 6.4, 6.7-6.8 of [1]).

Unit III

Respiratory Gas Flow: Introduction - Gas flow in the airway - Interaction between Convection and Diffusion - Exchange between Alveolar Gas and Erythrocytes - Ventilation/Perfusion Ratio (7.1 to 7.5 of]1]).

Unit IV

Basic Transport Equations According to Thermodynamics - Molecular Diffusion - Mechanisms in Membranes and Multiphasic Structure: Introduction - The laws of Thermodynamics - The Gibbs and Gibbs-Duhem Equations - Chemical Potential - Entropy in a system with Heat and Mass transfer - Diffusion, filtration, and Fluid movement in Interstitial Space from the point of view of Thermodynamics - Diffusion from the Molecular Point of view (8.1-8.7).

Text Book:

[1] Y.C.Fung, Biomechanics, Springer- Verlag, New York Inc., 1990

M.Sc.(Applied Mathematics)

Bio-Mechanics

AM453C | Paper IIIC | Semester IV

Practical Questions

1. Reduce the Navier-Stokes equations to theory of Elasticity.
2. Obtain mathematical model for the blood flow from Elastic Chamber.
3. Obtain volume flow rate $\dot{Q}$ for the steady laminar flow in an Elastic tube.
4. Obtain volume flow rate $\dot{Q}$ when the flow in Collapsible tubes and find $Q_{\max}$ value.
5. Obtain the mathematical model for the flow in an Interalveolar septum. Also classified Zones for alveolar blood flow.
6. Applications of Zone-2 in open and closed Capillary sheet.
7. Find the rate of energy dissipation in various airway tubes.
8. Obtain the measurement of diffusion capacity.
9. Applications of Gibbs and Gibbs-Duhem equations.
10. Applications of Chemical Potential.
11. Obtain Entropy production in a system with heat and mass transfer.
12. Obtain the diffusion coefficient when diffusion occurs from the molecular point of view.

Department of Mathematics
Osmania University
M.Sc Applied Mathematics

AM404 A Semester-IV

Functional Analysis
Paper-IV A

Unit –I

NORMED LINEAR SPACES: Definitions and Elementary Properties, Subspace, Closed Subspace, Finite Dimensional Normed LinearSpaces and Subspaces, Quotient Spaces, Completion of Normed Spaces.

Unit-II

HILBERT SPACES: Inner Product Space, Hilbert Space, Cauchy-Bunyakovsky-Schwartz Inequality, Parallelogram Law, Orthogonality, Orthogonal Projection Theorem, Orthogonal Complements, Direct Sum, Complete Orthonormal System, Isomorphism between Separable HilbertSpaces.

Unit-III

LINEAR OPERATORS: Linear Operators in Normed Linear Spaces, Linear Functionals, The Space of Bounded Linear Operators, Uniform Boundedness Principle,Hahn-Banach Theorem, Hahn-Banach Theorem for Complex Vector and Normed Linear Space, The General Form of Linear Functionals in Hilbert Spaces.

Unit-IV

FUNDAMENTAL THEOREMS FOR BANACH SPACES AND ADJOINT OPERATORS IN HILBERT SPACES: Closed Graph Theorem, Open Mapping Theorem, Bounded Inverse Theorem, Adjoint Operators, Self-Adjoint Operators, Quadratic Form, Unitary Operators, Projection Operators.

Text Book:

A First Course in Functional Analysis-Rabindranath Sen, Anthem Press An imprint of Wimbledon Publishing Company.

Reference:

1. Introductory Functional Analysis- E.Kreyzig- John Wilely and sons, New York,
2. Functional Analysis, by B.V. Limaye 2nd Edition.
3. Introduction to Topology and Modern Analysis- G.F.Simmons. Mc.Graw-Hill International Edition.

M.Sc. Applied Mathematics
Practicals Questions

AM 454 A

Semester-IV

Functional Analysis

Paper-IV A

1. Let ρ be the matric induce by a norm on a linear space $E \neq \phi$. If ρ_1 is defined by
$$\rho_1(x, y) = \begin{cases} 0 & x = y \\ 1 + \rho(x, y) & x \neq y \end{cases}$$
then prove that ρ_1 can't be obtain from a norm on E.

2. (a). Show that the closure $\bar{X}$ of a subspace X of a normed linear space E is again a subspace of E.
(b). Prove that the intersection of an arbitrary collection of non-empty closed subspaces of the normed linear space E is a closed subspace of E.

3. Let E_1 be a closed subspace and E_2 be a finite dimensional subspace of a normed linear space E. Then show that $E_1 + E_2$ is closed in E.

4. Show that a finite dimensional normed linear space is separable.

5. Show that equivalent norms on a vector space E induces the same topology on E.

6. Let C be a convex set in a Hilbert space H, and $d = inf\{\|x\|, x \in C\}$.If $\{x_n\}$ is a sequence in C such that $\lim_{x\to\infty}\|x_n\| = d$, show that $\{x_n\}$ is a Cauchy sequence.

7. Show that if M and N are closed subspaces of a Hilbert space H, then $M + N$ is closed provided $x \perp y$ for all $x \in M$ and $y \in N$.

8. Let $\{a_1, a_2, ...a_n\}$ be an orthogonal set in a Hilbert space H, and $\alpha_1, \alpha_2, ...\alpha_n$ be scalars such that their absolute values are respectively 1. Show that $\|\alpha_1 a_1 + ... + \alpha_n a_n\| = \|a_1 + a_2 + ... + a_n\|$.

9. Let H be a Hilbert space, $M \subseteq H$ a convex subset, and x_n a sequence in M such that $\|x_n\| \to d$ as $n \to \infty$ where $d = \inf_{x\in M} \|x\|$.Show that $\{x_n\}$ converges in H.

10. Let $x_1, x_2, ...x_n$ satisfy $x_i \neq 0$ and $x_i \perp x_j$ if $i \neq j$, $i, j = 1, 2, ...n$. Show that the $x_i's$ are linearly independent and extend the Pythagorean theorem from 2 to n dimensions.

11. Let E be a linear space over a scalar field $\mathbb{R}(or\mathbb{C})$. Prove that the space of continuous linear operators mapping E into itself is a ring.

12. Prove that every linear operator on a normed space is continuous iff bounded.

13. Give an example of an linear operator which is not bounded. Explain.

14. Let $x_0 \neq 0$ be a fixed element in a normed linear space E. Then prove that there exists a linear functional $f(x)$, defined on the entire space E, such that $\|f\| = 1$ and $f(x_0) = \|x_0\|$.

15. Let L be a closed linear subspace of a normed linear space E, and x_0 be a vector not in L. If d is the distance from x_0 to L, show that there exists a functional $f_0 \in E^*$ such that $f_0(L) = 0$, $f_0(x_0) = 1$ and $\|f_0\| = \frac{1}{d}$.

16. Given that E is a Banach space, $\mathcal{D}(T) \subseteq E$ is closed, and the linear operator T is bounded, show that T is closed.

17. Prove that for the projections P_1 and P_2 to be orthogonal, it is necessary and sufficient that the corresponding subspace L_1 and L_2 are orthogonal.

18. Give an example of a normal operator which is neither self-adjoint nor unitary. Explain.

19. Let $\|.\|$ and $\|.\|'$ be norms on a linear space E. Then prove that the norm $\|.\|$ is stronger than $\|.\|'$ if and only if there is some $\alpha > 0$ such that $\|x\| \leq \alpha \|x\|'$ for all $x \in E$.

20. Prove that P is a self-adjoint operator with its norm equal to one and P satisfies $P^2 = P$.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY

M.Sc. Applied Mathematics

AM –404 B Semester -IV

Discrete Mathematics

Paper-IV (B)

UNIT- I

LATTICES: Partial Ordering – Lattices as Posets – some properties of Lattices – Lattices as Algebraic Systems – Sublattices, Direct products and Homomorphisms – some special Lattices – Complete, complemented and distributive lattices.
(Pages 183-192, 378-397 of [1])

UNIT- II

BOOLEAN ALGEBRA: Boolean Algebras as Lattices – Bolean Identities – the switching Algebra – sub algebra, Direct product and homomorphism – Join irreducible elements – Atoms (minterms) – Boolean forms and their equivalence – minterm Boolean forms – Sum of products canonical forms – values of Boolean expressions and Boolean functions – Minimization of Boolean functions – the Karnaugh map method.
(Pages 397 – 436 of [1])

UNIT- III

GRAPHS AND PLANAR GRAPHS **:** Directed and undirected graphs – Isomorphism of graphs – subgraph – complete graph – multigraphs and weighted graphs – paths – simple and elementary paths – circuits – connectedness – shortest paths in weighted graphs – Eulerian paths and circuits – Incoming degree and outgoing degree of a vertex - Hamiltonian paths and circuits – Planar graphs – Euler's formula for planar graphs.
(Pages 137-159, 168-186 of [2])

UNIT- IV

TREES AND CUT-SETS: Properties of trees – Equivalent definitions of trees - Rooted trees – Binary trees – path lengths in rooted trees – Prefix codes – Binary search trees – Spanning trees and Cut-sets – Minimum spanning trees
(Pages 187-213 of [2])

Text Books:-

[1] J P Tremblay and R. Manohar: Discrete Mathematical Structures with applications to Computer Science, McGraw Hill Book Company
[2] C L Liu : Elements of Discrete Mathematics, Tata McGraw Hill Publishing Company Ltd. New Delhi. (Second Edition).

M.Sc. Applied Mathematics
Practicals Questions

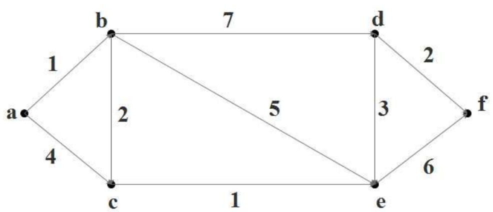
AM 454B

Semester-IV

Discrete Mathematics

Paper-IV (B)

1. Show that in a lattice if $a \leq b$ and $c \leq d$, then $a * c \leq b * d$.
2. Show that every interval of a lattice is a sublattice.
3. Prove that every chain is a distributive lattice.
4. Prove that the direct product of any two distributive lattice is a distributive lattice.
5. Show that Demorgan's laws hold in a complemented distributive lattice.
6. Write the boolean expression $x_1 \oplus x_2$ in an equivalent sum-of-products canonical form and product-of-sum canonical form in three variables x_1, x_2 and x_3.
7. Show that the following boolean expression are equivalent to one another (a) $(x \oplus y) * (x' \oplus z) * (y \oplus z)$.
(b) $(x * z) \oplus (x' * y) \oplus (y * z)$.
(c) $(x \oplus y) * (x' \oplus z)$.
(d) $(x * z) \oplus (x' * y)$.
8. Obtain simplified boolean expression which is equivalent to the expression $m_6 + m_7 + m_9 + m_{11} + m_{13}$.
9. Use the Karnaugh map representation to find a minimal sum-of-products expression of the function $f(a, b, c, d) = \sum(0, 1, 2, 3, 13, 15)$.
10. Expand the function $f(w, x, y, z) = w + \overline{y}z + \overline{x}y$ into their canonical sum-of-product form.
11. List all non-isomorphic simple graphs on 4 or fewer vertices.
12. Apply Dijkstras algorithm to determine a shortest path between a and f in the following weighted graph.



13. Give an example of a graph which has both an Eulerian circuit and Hamilton circuit.

14. Show that a complete directed graph always possesses a Hamiltonian.
15. Let G be a connected planar graph with ν vertices e edges where $\nu \geq 3$ then $e \leq 3\nu - 6$.
16. Built a binary search tree for a given sequence of number $17, 23, 4, 7, 9, 19, 45, 6, 2, 37, 99$.
17. How many different (pairwise non-isomorphic) trees are there of order 6 that is with 6 vertices.
18. Prove that an undirected graph with $e = \nu - 1$ that has no circuit is a tree.
19. Prove that every circuit has an even number of edges in common with every cut-set in a connected graph.
20. Prove that a tree with two or more vertices have at least two leaves.

OSMANIA UNIVERSITY

M.Sc. Applied Mathematics

AM 404C Semester IV

Topology

Paper IV(C)

UNIT I

Topological Spaces: The Definition and examples- Elementary concepts- Open bases and open subbases- Weak topologies.

UNIT II

Compactness: Compact spaces- Products of spaces- Tychonoff's theorem and locally compact spaces- Compactness for metric spaces- Ascoli's theorem.

UNIT III

Separation: T_1- spaces and Hausdorff spaces- Completely regular spaces and normal spaces- Urysohn's lemma and the Tietze extension theorem- The Urysohn imbedding theorem.

UNIT-IV

Connectedness: Connected spaces- The components of a spaces- Totally disconnected spaces- Locally connected spaces.

Text Books:

[1] Introduction to Topology and Modern Analysis (Chapters 3,4,5,6)

By G.F. Simmon's, Tota Mc Graw Hill Edition

M.Sc. Applied Mathematics

TOPOLOGY

AM 454C **Paper IV(C)** **Semester IV**

Practical Questions

1. Let T_1 and T_2 be two topologies on a non- empty X, then show that - $T_1 \cap T_2$ is also a topology on X. Does this hold for unions Justify.

2. Let x be a non- empty set and consider the class of subsets of X consisting of the empty set ϕ and all sets whose complements are countable. Is this a topology on X ?

3. Let X be a topological space. Then any closed subset of X is the disjoint of its boundary and its interior.

4. Let f:X ---►Y be a mapping of one topological space into another and let there be given an open base in X and open sub base with its generated open base in Y. Then f is continuous $\Longleftrightarrow$ the inverse image of each basic open set is open $\Longleftrightarrow$ the inverse image of each sub basic open set is open

5. Let f: X ---- Y be a mapping of one topological space into another. Then prove that f is open $\Longleftrightarrow$

   the image of each basic open set is open

6. Prove the converse of Heine – Borel Theorem. i.e. envy compact subspace of the real line is closed and bonded.

7. Show that a continuous real function of defined on a compact space X attains its infinium and its supremum.

8. Define product topology and give an example.

9. Show that compact metric space is separable.

10. Show that a closed subspace of a complete metric space is compact if and only if it is totally bounded.

11. Show that any finite T_1 – Space is discrete.

12. Show that a closed subspace of a normal space is normal

13. Let X be a T_1 – Space and show that X is normal if and only if each neighbourhood of a closed set F contains closure of some neighbourhood of F

14. Prove that every metric space is normal.

15. Show that a sub space of a completely regular space in completely regular.

16. Show that a topological space is connected if any only if Ø and X are the only subsets of X which are both open and closed.
17. Show that the space of all irrational numbers considered as a subspace of real line in totally disconnected.
18. Show that closure of any connected space is connected is connected
19. Show that a discrete space having only one point is connected and nay discrete space having more than one point is disconnected.
20. Define a Topology T on R such that (R, T) is disconnected.

Departemnt of MATHEMATICS,OU
Proposed Choice Based Credit System (CBCS)
M.Sc Mathematics

Semester -I

S.No.	Code No	Paper	Paper Title	Hrs/Week	Internal Assessment	Semester Exam	Total Marks	Credits
1. Core	MM 101	I	Algebra	4	20	80	100	4
2. Core	MM 102	II	Analysis	4	20	80	100	4
3. Core	MM 103	III	Mathematical Methods	4	20	80	100	4
4. Core	MM 104	IV	Elementary Number Theory	4	20	80	100	4
5. Practical	MM 151	Practical	Algebra	4	….	50	50	2
6. Practical	MM 152	Practical	Analysis	4	….	50	50	2
7. Practical	MM 153	Practical	Mathematical Methods	4	….	50	50	2
8. Practical	MM 154	Practical	Elementary Number Theory	4	….	50	50	2
			Total :	**32**				**24**

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. Mathematics

Algebra

MM 101 Paper I Semester I

Unit I

Automaphisms- Conjugacy and G-sets- Normal series solvable groups- Nilpotent groups. (Pages 104 to 128 of [1])

Unit II

Structure theorems of groups: Direct product- Finitly generated abelian groups- Invariants of a finite abelian group- Sylow's theorems- Groups of orders p^2,pq . (Pages 138 to 155)

Unit III

Ideals and homomsphism- Sum and direct sum of ideals, Maximal and prime ideals- Nilpotent and nil ideals- Zorn's lemma (Pages 179 to 211).

Unit-IV

Unique factorization domains - Principal ideal domains- Euclidean domains- Polynomial rings over UFD- Rings of traction.(Pages 212 to 228)

Text Books:

[1] Basic Abstract Algebra by P.B. Bhattacharya, S.K. Jain and S.R. Nagpanl.

Reference: 1] Topics in Algebra by I.N. Herstein.

M.Sc. (Mathematics)

Algebra

MM 151 **Paper I** **Semester I**

Practical Questions

1. A finite group G having more than two elements and with the condition that $x^2 \neq e$ for some $x \in G$ must have nontrivialautomorphism.
2. (i) Let G be a group Define $a * x = ax,\ a, x \in G$ then the set G is a G-set

   (ii) Let G be a group Define $a * x = axa^{-1}\ a, x \in G$ then G is a G-set.
3. An abelian group G has a composition series if and only if G is finite
4. Find the number of different necklaces with p beads p prime where the beads can have any of n different colours
5. If G is a finite cyclic group of order n then the order of Aut$G$, the group of automorphisms of G, is $\phi(n)$, where ϕ is Euler's function.
6. If each element $\neq e$ of a finite group G is of order2 then $|G| = 2^n$ and $G \approx C_1 \times C_2 \times \ldots\ldots \times C_n$ where C_i are cyclic and $|C_i| = 2.$
7. (i) Show that the group $\frac{Z}{<10>}$ is a direct sum of $H = \{\bar{0}\ \bar{5}\}$ *and* $K = \{\bar{0}\ \bar{2}\ \bar{4}\ \bar{6}\ \bar{8}\}$

   (ii) Show that the group $\left(\frac{z}{\langle 4 \rangle}, +\right)$ cannot be written as the direct sum of two Subgroups of order 2.
8. (i) Find the non isomorphic abelian groups of order 360

   (ii) If a group of order p^n contains exactly one sub group each of orders $p, p^2, \_\ \_\_ P^{n-1}$ then it is cyclic.
9. Prove that there are no simple groups of orders 63, 56, and 36
10. Let G be a group of order 108. Show that there exists a normal subgroup of order 27 or 9.
11. (i) Let **R** be acommutative Ring with unity. Suppose R has no nontrivial ideals .Prove that R is a field.

    (ii) Find all ideals in Z and in $\frac{Z}{\langle 10 \rangle}$

12. (i) The only Homomorphism from the ring of integers Z to Z are the identity and Zero Mappings.
    (ii) Show that any nonzero homomorphism of a field F into a ring R is one-one.
13. For any tow ideals A and B in a Ring R (i) $\frac{A+B}{B} \approx \frac{A}{A \bigcap B}$
    (ii) $\frac{A+B}{A \cap B} \approx \frac{A+B}{A} \times \frac{A+B}{B} \approx \frac{B}{A \cap B} \times \frac{A}{A \cap B}$ In particular if $R = A + B$ then $\frac{R}{A \cap B} \approx \frac{R}{A} \times \frac{R}{B}$.
14. Let R be a commutative ring with unity in which each ideal is prime then R is a field
15. Let R be a Boolean ring then each prime ideal $P \neq R$ is maximal.
16. The commutative integral domain $R = \{a + b\sqrt{-5} / a, b \in Z\}$ is not a UFD.
17. (i) The ring of integers Z is a Euclidean domain
    (ii) The Ring of Gausion Integers $R = \{m + n\sqrt{-1} / m, n \in Z\}$ is a Euclidean domain
18. (i) Prove that $2 + \sqrt{-5}$ is irreducible but not prime in $Z(\sqrt{-5})$
    (ii) Show that $1 + 2\sqrt{-5}$ *and* 3 are relatively prime in $Z(\sqrt{-5})$
19. Let R be a Euclidean domain . Prove the following
    (i) If $b \neq 0$ *then* $\phi(a) < \phi(b)$
    (ii) If a and b are associates then $\phi(a) = \phi(b)$
    (iii) If a/b *and* $\phi(a) = \phi(b)$ *then* a and b are associates
20. Prove that every nonzero prime ideal in a Euclidean domain is maximal.

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. Mathematics

MM – 102 Semester I

Analysis

Paper-II

Unit I

Metric spaces- Compact sets- Perfect sets- Connected sets

Unit II

Limits of functions- Continuous functions- Continuity and compactness Continuity and connectedness- Discontinuities – Monotone functions.

Unit III

Rieman- Steiltjes integral- Definition and Existence of the Integral- Properties of the integral- Integration of vector valued functions- Rectifiable waves.

Unit-IV

Sequences and series of functions: Uniform convergence- Uniform convergence and continuity- Uniform convergence and integration- Uniform convergence and differentiation- Approximation of a continuous function by a sequence of polynomials.

Text Books:

[1] Principles of Mathematical Analysis (3rd Edition) (Chapters 2, 4, 6) By Walter Rudin, Mc Graw-Hill Internation Edition.

M.Sc. Mathematics

Analysis

MM 152 Paper –II Semester –I

Practical Questions

1. Construct a bounded set of real numbers with exactly three limit points
2. Suppose E^1 is the set of all limit points of E. Prove that E^1 is closed also prove that E and E have the same limit points.
3. Let E^0 demote the set of all interior points of a set E. Prove that E^0 is the largest open set contained in E Also prove that E is open if and only if $E = E^0$
4. Let Xbe an infinite set.For $p \in X$, $q \in X$ define

$$d(p,q) = \begin{cases} 1 & if\ p \neq q \\ 0 & if\ \ p = q \end{cases}$$

Prove that this is a metric, which subsets of the resulting metric space are open, which areclosed? Which are compact?

5. i) If A and B are disjoint closed sets in some metric space X, prove that they are separated
   ii) Prove the same for disjoint open sets
   iii)Fixa $p \in X$ and $\delta > o$, Let A = { $q \in X : d(p,q) < \delta$}

and $B = \{q \in X : d(p,q) > \delta\}$ prove that A and B are separated.

6. i) Suppose f is a real function on R which satisfies $\lim\limits_{h \to o}[f(x+h) - f(x-h) = o]$ for every $x \in R$ Does this imply that f is continuous? Explain

   ii) Let f be a continuous real function on a metric space X,let Z(f) = $\{p \in X : f(p) = 0\}$ prove that z (f) is closed.

7. If f is a continuous mapping of a metric space X into a metric space Y .prove that

$$f(\overline{E}) \subset \overline{f(E)} \quad \text{for every set } E \subset X$$

8. Let f and g be continuous mapping of a metric space X into a metric space Y Let E be a dense subset of X. Prove that
   i) f(E) is dense in f(X)
   ii) If g(p) =f(p) $\forall$ p $\in$ E , Prove that g(p) =f(p) $\forall$ p $\in$ X

9. Suppose f is a uniformly continuous mapping of a metric space X into a metric space Y and { X_m} is a Couchy sequence in X prove that {f(X_m)} is Cauchy sequence in Y

10. Let I = [0 , 1] be the closed unit interval, suppose f is a continuous mapping of f into I. Prove that f(x) = x for at least one x

11. Suppose α increases on [a , b] , a <x_o<b, α is continuous at $x_{0,}$ $f(x_0) = 1$ and f(x) =0 if x $\neq$ x_o .Prove that f $\in$ R(α) and $\int_a^b f\,d\alpha = 0$

12. Suppose f $\geq$0 and f is continuous on [a , b] and $\int_a^b f(x)dx = 0$, Prove that f(x) =0 $\forall$ x€ [a , b]

13. If f(x) = 1or 0 according as x is rational or not .Prove that f $\notin R$ on [a , b] for any a,b,€R with a<b.Also prove that f $\notin$ R(α) on [a , b] with respect to any monotonically increasing function α on [a , b]

14. Suppose f is a bounded real function on [a , b] and f^2€R on [a , b]. Does it follow that $f \in R$?

    Does the answer change if we assume that f^3€R?

15. Suppose γ_1 *and* γ_2 are the curves in the complex plane defined on [0,2π] by $\gamma_1(t)= e^{it}$, $\gamma_2(t) = e^{2it}$

    Show that the two curves have the same range

    Also Show that γ_1 *and* γ_2 are rectifiable and find the curve length of γ_1 *and* γ_2

16. Discuss the uniform conversance of the sequence of functions $\{f_n\}$ where

$$f_n(x) = \frac{\sin nx}{\sqrt{n}} \quad \text{x real, n = 1,2,3....}$$

17. Give an example of a series of continuous functions whose sum function may be discontinuous.
18. Discuss the uniform conversance of the sequence

$$f_n(x)=\frac{1}{1+nx} \; x > 0,\ n = 1,2,3...$$

19. Give an example of a sequence of functions such that

$$\lim \int f_n \neq \int \lim f_n$$

20. Prove that a sequence $\{f_n\}$ converse to f with respect to the metric of C(x) if and only if $f_n \to f$ uniformly on X

M.Sc. Mathematics
Mathematical Methods

MM 153 Paper III Semester I

Practical Questions

1. Compute the first three successive approximations for the solution of the initialvalue problem $\frac{dx}{dt} = x^2, \quad x(0) = 1.$

2. Solve $yp = 2yx + \log q.$

3. Solve $yzp + zxq = xy$ with usual notations.

4. Explain Strum-Liouille's boundary value problems.

5. Classify the equation $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0.$

6. Solve $r + t + 2s = 0$ with the usual notations.

7. Find the particular integral of the equation $(D^2 - D)Z = e^{2x+y}.$

8. Solve in series the equation $xy'' + y' - y = 0.$

9. Solve $y'' - y = x$ using power series method.

10. Solve the Froenius method $x^2 y'' + 2x^2 y' - 2y = 0.$

11. Solve in series $2xy'' + 6y' + y = 0.$

12. Prove that $J_{-n}(x) = (-1)^n J_n(x)$ where n is an integer.

13. Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x).$

14. Prove that $H_n(-x) = (-1)^n H_n(x).$

15. Show that $H_{2n+1}(0) = 0$.

16. Show that $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$.

17. Solve $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$; *with* $u(x,0) = 4e^{-x}$ using separation of variable method.

18. Find the surface passing through the parabolas $Z = 0,\ y^2 = 4ax$ *and* $Z = 1,\ y^2 = -4ax$ and satisfying the equation $xr + zp = 0$.

19. Find the surface satisfying $t = 6x^2y$ containing two lines y = 0 = z and y = 2 = z .

20. Reduse the equation $x^2r - y^2t + px - qy = x^2$ in the canonical form.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY

M.Sc. Mathematics
Semester I

MM104 Elementary Number Theory

Paper- IV

UNIT-I

The Division Algorithm- Number Patterns- Prime and Composite Numbers- Fibonacci and Lucas' numbers- Fermat Numbers- GCD-The Euclidean Algorithm- The Fundamental Theorem of Arithmetic- LCM- Linear Diophantine Equations

UNIT-II

Congruences- Linear Congruences- The Pollard Rho Factoring Method- Divisibility Tests- Modular Designs- Check Digits- The Chinese Remainder Theorem- General Linear Systems- 2X2 Systems

UNIT-III

Wilson's Theorem- Fermat's Little Theorem- Pseudo primes- Euler's Theorem- Euler's Phi function Revisisted- The Tau and Sigma Functions- Perfect Numbers- Mersenne Primes- The Mobius Function

UNIT-IV

The Order of a Positive Integer- Primality Tests- Primitive Roots for Primes- Composites with Primitive roots- The Algebra of Indices- Quadratic Residues- The Legendre Symbol- Quadratic Reciprocity- The Jacobi Symbol

Text Book : Thomas Koshy , *Elementary Number Theory with Applications*

Elementary Number theory

MM154

Practicals Question Bank

Semester I

UNIT-I

1

Find the positive integer a if $[a, a+1] = 132$.

2

Find the twin primes p and q such that $[p, q] = 323$.

3

The LDE $ax + by = c$ is solvable if and only if $d|c$, where $d = (a, b)$. If x_0, y_0 is a particular solution of the LDE, then all its solutions are given by

$$x = x_0 + \left(\frac{b}{d}\right)t \quad \text{and} \quad y = y_0 - \left(\frac{a}{d}\right)t$$

where t is an arbitrary integer.

4

Solve the LDE $1076x + 2076y = 3076$ by Euler's method.

5

Find the general solution of each LDE

$2x + 3y = 4$

$12x + 13y = 14$

UNIT-II

6

Determine the number of incongruent solutions of each linear congruence.

$12x \equiv 18 \pmod{15}$

$28u \equiv 119 \pmod{91}$

$49x \equiv 94 \pmod{36}$

7

Using congruences, solve each LDE.

$3x + 4y = 5$

$15x + 21y = 39$

8

Using the Pollard rho method, factor the integer 3893.

9

Prove that the digital root of the product of twin primes, other than 3 and 5, is 8.

10

Using the CRT, solve Sun-Tsu's puzzle:

$$x \equiv 1 \pmod 3, \quad x \equiv 2 \pmod 5, \quad \text{and} \quad x \equiv 3 \pmod 7$$

UNIT-III

11

Prove each, where p is a prime.

Let p be odd. Then $2(p-3)! \equiv -1 \pmod{p}$.
$(p-1)(p-2)\cdots(p-k) \equiv (-1)^k k! \pmod{p}$, where
$1 \le k < p$.

12

Find the remainder when 24^{1947} is divided by 17.

13

Let p be any odd prime and a any nonnegative integer.
Prove the following.

$1^{p-1} + 2^{p-1} + \cdots + (p-1)^{p-1} \equiv -1 \pmod{p}$
$1^p + 2^p + \cdots + (p-1)^p \equiv 0 \pmod{p}$

14

Verify each.

$(12+15)^{17} \equiv 12^{17} + 15^{17} \pmod{17}$
$(16+21)^{23} \equiv 16^{23} + 21^{23} \pmod{23}$

15

Find the remainder when 245^{1040} is divided by 18.

Unit IV

16

Evaluate $(-4/41)$ and $(-9/83)$.

17

Verify that $9973 | (2^{4986} + 1)$.

18

Prove that there are infinitely many primes of the form $4n+1$.

19

Show that $1! + 2! + 3! + \cdots + n!$ is never a square, where $n > 3$.

20

Prove that there are infinitely many primes of the form $10k - 1$.

DEPARTMENT OF MATHEMATICS,OU
Proposed Choice Based Credit System (CBCS)
M.Sc Mathematics

Semester -II

S.No.	Code No	Paper	Paper Title	Hrs/Week	Internal Assessment	Semester Exam	Total Marks	Credits
1. Core	MM 201	I	Advnaced Algebra	4	20	80	100	4
2. Core	MM 202	II	Advnaced Analysis	4	20	80	100	4
3. Core	MM 203	III	Theory of Ordinary differential equation	4	20	80	100	4
4. Core	MM 204	IV	Topology	4	20	80	100	4
5. Practical	MM 251	Practical	Advanced Algebra	4	….	50	50	2
6. Practical	MM 252	Practical	Advnaced Analysis	4	….	50	50	2
7. Practical	MM 253	Practical	Theory of Ordinary differential equation	4	….	50	50	2
8. Practical	MM 254	Practical	Topology	4	….	50	50	2
			Total :	**32**				**24**

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. (Mathematics)

MM –201 Semester II

Advanced Algebra

Paper I

Unit I

Algebraic extensions of fields: Irreducible polynomials and Eisenstein criterion- Adjunction of roots- Algebraic extensions-Algebraically closed fields (Pages 281 to 299)

Unit II

Normal and separable extensions: Splitting fields- Normal extensions- Multiple roots- Finite fields- Separable extensions (Pages 300 to 321)

Unit III

Galois theory: Automorphism groups and fixed fields- Fundamental theorem of Galois theory- Fundamental theorem of Algebra (Pages 322 to 339)

Unit-IV

Applications of Galoes theory to classical problems: Roots of unity and cyclotomic polynomials- Cyclic extensions- Polynomials solvable by radicals- Ruler and Compass constructions. (Pages 340-364)

Text Books:

[1] Basic Abstract Algebra- S.K. Jain, P.B. Bhattacharya, S.R. Nagpaul.

Reference Book: Topics in Algrbra B y I. N. Herstein

M.Sc. Mathematics

Advanced Algebra

MM 251 **Paper I** **Semester II**

Practical Questions

1. (i) $\phi_p(x)=1+x+.....+x^{p-1}$ is irreducible over Q. Where p is a prime.

   (ii) Show that $x^3+3x+2\in\frac{Z}{\langle 7\rangle}(x)$ is irreducible over the field $\frac{Z}{\langle 7\rangle}$.

2. Show that the following polynomials are irreducible over Q

   (i) x^3-x-1 (ii) x^4-3x^2+9 (iii) x^4+8

3. Show that there exists an extension of $E\ of\ \frac{Z}{\langle 3\rangle}$ with nine elements having all the roots of $x^2-x-1\in\frac{Z}{\langle 3\rangle}(x)$

4. (i) Show that there is an extension E of R having all the roots of $1+x^2$

   (ii) Let $f_i(x)\in F(x)$ for i= 1, 2, ...m then there exists an extension E of F in which each polynomial has root

5. Show that $\sqrt{2}\ and\ \sqrt{3}$ are algebraic over Q and find the degree of $Q(\sqrt{2})$ over Q and $Q(\sqrt{3})$ over Q.

   (iii) Find a suitable number a such that $Q(\sqrt{2},\sqrt{5})=Q(a)$.

6. Show that the degree of the extension of the splitting field of $x^3-2\in Q(x)$ is 6

7. Let p be a prime then $f(x)=x^p-1\in Q(x)$ has a splitting field $Q(\alpha)$ where $\alpha\neq 1$ and $\alpha^p=1$. Also $[Q(\alpha):Q]=p-1$

8. Show that the splitting field of $f(x)=x^4-2\in Q(x)\ over\ Q\ is\ Q(2^{\frac{1}{4}},i)$ and its degree of extension is 8

9. If the multiplicative group F^* of non zero elements of a field F is cyclic then F is Finite

10. The group of automorphisms of a field F with p^n elements is cyclic of order n and generated by ϕ where $\phi(x)=x^p,\ x\in F$

11. The group $G(\frac{Q(\alpha)}{Q})$ where $\alpha^5=1\ and\ \alpha\neq 1$ is isomorphic to the cyclic group of order 4

12. Let $E = Q(\sqrt[3]{2}, \omega)\, where\, \omega^3 = 1, \omega \neq 1$ let σ_1 be the identity automorphism of E and Let σ_2 be an automorphism of E such that $\sigma_2(\omega) = \omega^2$ and $\sigma_2(\sqrt[3]{2}) = \omega(\sqrt[3]{2})$. If $G = \{\sigma_1, \sigma_2\}$ then $E_G = Q(\sqrt[3]{2}\omega^2)$
13. If $f(x) \in F(x)$ has r distinct roots in its splitting field E over F then the Galois group $G\left(\frac{E}{F}\right) of\ f(x)$ is a subgroup of the symmetric group $S_{r.}$
14. The Galois group of $x^4 - 2 \in Q(x)$ is the octic group.
15. The Galois group of $x^4 + 1 \in Q(x)$ is Klein four group
16. $\phi_8(x)$ and $x^8 - 1$ have the same Galois group namely $\left(\frac{Z}{\langle 8 \rangle}\right)^* = \{1,3,5,7\}, the$ Klein's four group.
17. If a field F contains a primitive n^th root of unity then the characteristic of F is Zero or a prime P that does not divide n
18. Show that the following polynomials are not solvable by radicals over Q
    (i) $x^7 - 10x^5 + 15x + 5$ (ii) $x^5 - 9x + 3$ (iii) $x^5 - 4x + 2$

19. It is impossible to construct a cube with a volume equal to twice the volume of a given cube by using ruler and compass only.

20. A regular n-gon is constructible if and only if $\phi(n)$ is a power of 2.

(equivalently the angle $\frac{2\pi}{n}$ is Constructible.)

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. Mathematics

MM –202 **Semeste II**

Advanced Analysis
Paper II

Unit I

Algebra of sets- Borel sets- Outer measure- Measurable sets and Lebesgue measure- A non-measurable set- Measurable functions- Little word's three principles.

Unit II

The Rieman integral- The Lebesgue integral of a bounded function over a set of finite measure- The integral of a non-negative function- The general Lebesgue integral.

Unit III

Convergence in measure- Differentiation of a monotone functions- Functions of bounded variation.

Unit-IV

Differentiation of an integral- Absolute continuity- The L^p-spaces- The Minkowski and Holder's inequalities- Convergence and completeness.

Text Books:[1] Real Analysis (3rd Edition) (Chapters 3, 4, 5)

by

H. L. Royden

Pearson Education (Low Price Edition)

M.Sc.Mathematics

Advanced Analysis

Paper II

MM252 Practical Questions Semester II

1. (i). Prove that the interval [0,1] is not countable.

   (ii). If A is the set of all irrational numbers in [0,1]. Prove that $m^*(A) = 1$.

2.(i). If $m^*(A) = 0$. Prove that $m^*(A \cup B) = m^*(B)$.

   (ii). Prove that if a σ −algebra of subsets of $\mathbb{R}$ contains intervals of the form (a, ∞) then it contains all intervals.

3. Show that a set E is measurable if and only if for each $\epsilon > 0$ there exists a closed set F and an open set O such that $F \subseteq E \subseteq O$ and $m^*(O - F) < \epsilon$.

4. (i). Show that if E_1 and E_2 are measurable then $m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$

   (ii). Suppose $\{A_k\}$ is an ascending collection of measurable sets. Prove that

   $m(\cup_{k=1}^{\infty} A_k) = \lim_{k\to\infty} m(A_k)$

5. Suppose A and B are any sets. Prove that

   (i). $\chi_{A\cap B} = \chi_A \chi_B$

   (ii). $\chi_{A\cup B} = \chi_A + \chi_B - \chi_A \chi_B$

   (iii). $\chi_{A^c} = 1 - \chi_A$

6. Let E have measure zero. Show that if f is a bounded function on E then f is measurable and $\int_E f = 0$.

7. Let $\{f_n\}$ be a sequence of non negative measurable functions that converge to f pointwise on E. Let $M \geq 0$ be such that $\int_E f_n \leq M$ for all n. Show that $\int_E f \leq M$.

8. Let f be a non negative measurable functions on E.

   Prove that $\int_E f = 0$ if and only if $f = 0$ a.e on E.

9. Let $\{f_n\}$ be a sequence of non negative measurable functions on E that converges pointwise on E to f. Suppose $f_n \leq f$ on E for each n, show that $\lim_{n\to\infty} \int_E f_n = \int_E f$.

10. Suppose $\{f_n\}$ is a sequence of measurable functions on E that converges pointwise on a.e. on E to f.

Suppose there is a sequence $\{g_n\}$ of non negative measurable functions on E that converges pointwise on a.e. on E to g and dominates $\{f_n\}$ on E in the sense that $|f_n| \leq g_n$ on E $\forall n$.

If $\lim_{n\to\infty} \int_E g_n = \int_E g$ prove that $\lim_{n\to\infty} \int_E f_n = \int_E f$.

11. Prove that pointwise convergence implies convergence in measure.

12. Construct a sequence of measurable functions which converges in measure but not point wise.

13. Suppose f, g are functions of bounded variation in [a,b]. Show that $f + g$ and λf for any scalar λ are also functions of bounded variation on $[a, b]$.

Also prove that i. $\tau_a^b(f + g) \leq \tau_a^b(f) + \tau_a^b(g)$

ii. $\tau_a^b(\lambda f) = |\lambda|\tau_a^b(f)$

14. Prove that the greatest integer function is a function of bounded variation on $[a, b]$

15. Show that continuous and bounded variation of a function are two independent concepts.

16. Show that the sum and difference and product of two absolutely continuous functions are also absolutely continuous.

17. Let f be absolutely continuous on $[c, d]$ and g be absolutely continuous on $[a, b]$with $c \leq g \leq d$. Prove that fog is absolutely continuous on $[a, b]$

18. Suppose f is absolutely continuous on $[a, b]$ and $E = \{x: f'(x) = 0\}$. Prove that $m\big(g(E)\big) = 0$

Note. f is absolutely continuous on E with $f'(x) = 0$ $\forall x \in E$ implies $f$is constant on E which implies $m\big(f(E)\big) = 0$

19. Let g be an absolutely continuous monotone function on [0,1] and E is a set of measure zero. Prove that $g(E)$ has measure zero.

20. (i). Show that $\|f + g\|_\infty \leq \|f\|_\infty + \|g\|_\infty$

(ii). If $f \in L^p$, $g \in L^p$ then prove that $f + g \in f \in L^p$

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY

M.Sc.-Mathematics

MM-203 Semester II

Theory of Ordinary Differential Equations

Paper-III

UNIT-I

Linear differential equations of higher order: Introduction-Higher order equations-A Modelling problem – Linear Independence- Equations with constant coefficients Equations with variable coefficients- Wronskian-Variation of parameters- Some
Standard methods.

UNIT-II

Existence and uniqueness of solutions: Introduction -preliminaries-successive approximations – Picard's theorem – continuation and dependence on intial conditions – existence of solutions in the large – existence and uniqueness of solutions of systems-fixed point method.

UNIT-III

Analysis and methods of non-linear differential equations:-Introduction – Existence theorem –Extremal solutions-Upper and Lower solutions-Monotone iterative method and method of quasi linearization- Bihari's inequality, Application of Bihari's inequality

UNIT-IV

Oscillation theory for linear Differential Equation of Second order:- The adjoint equation-Self adjoint linear differential equation of second order-Abel's formula- the number of zeros in a finite interval- The sturm separation theorem- the sturm comparison theorem –the sturm picone the Bocher Osgood theorem-A special pair of solution-Oscillation on half axis.

Text Book :

1) Text book of Ordinary Diffferential Equations
   by S.G.Deo, V. Lakshmikantham, V.Raghavendra

2) An Introduction to the Theory of Ordinary Differential Equations
   by V. Dharmaiah, , PHI Publishers.

3) An Introduction to the theory of Ordinary Differential Equation
   by Walter Leighton

M.Sc. Mathematics
Theory of Ordinary differential equation
Paper III

MM 253 **Semister II**

Practical Questions

1. Define the functions f and g on [-1,1] by $\left.\begin{array}{l} f(x)=0 \\ g(x=1) \end{array}\right\} if \;\; x \in [-1,\, 0];$ $\left.\begin{array}{l} f(x)=\sin x \\ g(x)=1-x \end{array}\right) if \;\; x \in [0,1]$ then prove that f and g are linearly independent on [-1 ,1].
2. Solve Euler equitation by assuming a solution of the form $x(t)=t^r$
   $(i) \;\; t^2x''+5tx'+4x=0$
   $(ii) \; t^2x''-9tx'+25x=0$
3. (a) Show that the equation $x''+px'+qx=r$, where p , q and r are continuous function on R and p' exists on R , reduce to $\frac{d^2f}{dt^2}+Qf=R$ by the transformation

$$x(t)=\exp[-\frac{1}{2}\int Pdt]f(t) \;\; where \; Q=q-\frac{1}{2}p'-\frac{1}{4}p^2 \;\; and \;\; R=r\;\exp\left[\frac{1}{2}\int Pdt\right]$$

   (b) Solve $x''+(\cot t)x'+6x=0$ given that one of the solution is $2-3\sin^2 t$

   (c) Find the solution of the equation $x''-x=1$ which vanishes when $t=0$ and tends to a finite limit as $t\to\infty$

4. (a) Consider the equitation $t^2x''+tx'-x=t$. Show that $\phi_1(t)=t \; and \; \phi_2(t)=\frac{-1}{2}t$ are solutions of the homogeneous equation. Use the variation of parameters method to show that a solution of the given equation is $x(t)=\frac{t^3}{8}+C_1t-\frac{C_2}{2t}$

   (b) Use the method of variation of Parameter to find a general solution of
   $(i) \;\; x'''-x'=\cos t$ (ii) $x'''-x'=e^{-t}$

5. (a) Find the particular solution by using the method of undetermined coefficient of $(i)\ x'' + 25x = 2\sin 2t$ $\quad (ii)\ x''' + 4x = 2\sin t + 1 + 3t^2 + 4e^t$

   (b) Consider the equitation $(1-t^2)x'' - 2tx' + 2x = 0, 0 < t < 1$ Given that $\phi_1(t) = t$ is a solution, Find the second linearly independent solution.

6. Find the Lipschitz constant and a bound for $f(t,x)$ in the region indicated. Also find an interval of local existence of solutions of IVP $x' = f(t,x), x(t_0) = x_0$ for the following problems

$$(i)\ f(t,x) = t\sin x,\ x(0) = 0,\ |t| \le \lambda,\ |x| \le 1 \quad (ii)\ f(t,x) = e^t(1+x)^{-1},\ x(0) = 0,\ |x| \le \frac{3}{4}, |t| \le 1$$

7. Show that the following function satisfying the Lipschitz condition in the rectangle indicated and find the Lipschitz constant.

$$(i)\ f(t,x) = (x + x^2)\frac{\cos t}{2},\ |x| \le 1,\ |t-1| \le \frac{1}{2} \qquad (ii)\ f(t,x) = \sin(xt);\ |x| \le 1,\ |t| \le 1$$

8. Show that the following function do not satisfy theLlipschitz condition in the region indicated $\quad (i)\ f(t,x) = \frac{e^t}{x^2},\ f(t,\ 0) = 0, |x| \le \frac{1}{2}\ , |t| \le 2$

   $(ii)\ f(t,n) = \frac{\sin n}{t},\ f(0,x) = 0, |x| \le \infty, |t| \le 1$
9. Calculate the successive approximation for IVPs
   $(i)\ x' = g(t)\ ; x(0) = 0$ $\quad (ii)\ x' = x\ , x(0) = 1$ $\quad (iii)\ x' = tx,\ x(0) = 1$
10. Consider the IVP $x' = t^2 + x^2\ , x(0) = 0, 0 \le t \le a,\ |x| \le b$ Show that
    (i) Solution $x(t)\, exists\ \ at\ 0 \le t \le \min\left(a, \frac{b}{a^2 + b^2}\right)$
    (ii) The maximum value of $\frac{b}{a^2 + b^2}\ is\ \frac{1}{2a}$ for a fixed a
    (iii) $h = \min(a, \frac{1}{2a})$ is largest when $a = \frac{1}{\sqrt{2}}$
11. Deduce Gronwall's inequality from Bihari's in equality.

12. Let the function f,v,g be defined as $f,v \in C[R^+,R^+]$ $g \in C[(0,\infty);(0,\infty)]$ *and* $g(x)$ be non decreasing in x and be sub additive, i.e.,

$g(u,v) \le g(u)+g(v), h \in C[R^+,R^+]$ and if $f(t) \le h(t)+\int_{t_0}^{t} v(s)g(f(s))ds,\ t \ge t_0$ then

Show that $f(t) \le h(t)+G^{-1}\left[G(c)+\int_{t_0}^{t} v(t)dt\right],\ t_0 \le t \le T$ Where C is a Constant and

G, G^{-1}, T are as given in Bihari's Inequality.

13. State and Prove Application of Bihari's Inequality.
14. Find Upper and Lower Solution of IVP's

$(i)\ x' = x^{\frac{1}{2}}, x(0)=0, t \ge 0 \qquad (ii)\ x' = x^2, x(0) = -1$

15. Find the minimal and maximal solution of IVPs

$(i)\ x' = 3x^{\frac{2}{3}}, x(0=0) \qquad (ii)\ x' = x^{\frac{1}{2}}, x(0)=0$

16. Represent following equation in to self adjoint form

$(i)\ y'' - 3y' + 2y = 0 \qquad (ii)\ x^2y'' + xy' + (x^2 - n^2)y = 0$

$(iii)\ (1-x^2)y'' - 2xy' + (n^2+n)y = 0$

17. Find a function $Z(x)$ such that

$(i)\ Z(x)(y''+y) = \frac{d}{dx}(k(x)y' + m(x)y) \qquad (ii)\ Z(x)(y''+3y'+2y) = \frac{d}{dx}(k(x)y' + m(x)y)$

18. (a) Given (n+1) is a solution of $y'' - 2(x+1)y' + 2y = 0$,Find the general Solution

(b) Find the general solution of differential equation $x^2y'' - xy' + y = 0$ by guessing one of its Solution

19. Which of the following differential equation possess more rapidly oscillating solution in the interval $(1,\infty)$

$(i)\ x^2y'' - xy' + y = 0,\ y'' + y = 0 \qquad (ii)\ y'' + (x+1)y = 0,\ y'' + (\sqrt{(h+x^2)})y = 0$

20. (a) Show that all solution of $x^p y'' + k^2 y = 0 (p > 0, K^2 > 0\, are\ the\ real\ cons\tan ts)$ vanish infinitely often an $(1,\infty)$ *iff* $p \le 2, K^2 \le \frac{1}{n}$

(b) Show that all solution of equation $(x^{-q}y')' + x^q y = 0$ $(q > 0$ a real constant) are Oscillatory on $(1,\infty)$

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. Mathematics

MM 204 Semester II

Topology

Paper IV

UNIT I

Topological Spaces: The Definition and examples- Elementary concepts- Open bases and open subbases- Weak topologies.

UNIT II

Compactness: Compact spaces- Products of spaces- Tychonoff's theorem and locally compact spaces- Compactness for metric spaces- Ascoli's theorem.

UNIT III

Separation: T_1- spaces and Hausdorff spaces- Completely regular spaces and normal spaces- Urysohn's lemma and the Tietze extension theorem- The Urysohn imbedding theorem.

UNIT-IV

Connectedness: Connected spaces- The components of a spaces- Totally disconnected spaces- Locally connected spaces.

Text Books:

[1] Introduction to Topology and Modern Analysis (Chapters 3,4,5,6)

By

G.F. Simmon's

Tota Mc Graw Hill Edition

TOPOLOGY

MM 254 **Paper IV** **Semester II**

Practical Questions

1. Let T_1 and T_2 be two topologies on a non- empty X, then show that - $T_1 \cap T_2$ is also a topology on X. Does this hold for unions Justify.

2. Let x be a non- empty set and consider the class of subsets of X consisting of the empty set ϕ and all sets whose complements are countable. Is this a topology on X ?

3. Let X be a topological space. Then any closed subset of X is the disjoint of its boundary and its interior.

4. Let f:X ---►Y be a mapping of one topological space into another and let there be given an open base in X and open sub base with its generated open base in Y. Then f is continuous $\Longleftrightarrow$ the inverse image of each basic open set is open $\Longleftrightarrow$ the inverse image of each sub basic open set is open

5. Let f: X ---- Y be a mapping of one topological space into another. Then prove that f is open $\Longleftrightarrow$ the image of each basic open set is open

6. Prove the converse of Heine – Borel Theorem. i.e. envy compact subspace of the real line is closed and bonded.

7. Show that a continuous real function of defined on a compact space X attains its infinium and its supremum.

8. Define product topology and give an example.

9. Show that compact metric space is separable.

10. Show that a closed subspace of a complete metric space is compact if and only if it is totally bounded.

11. Show that any finite T_1 – Space is discrete.

12. Show that a closed subspace of a normal space is normal

13. Let X be a T_1 – Space and show that X is normal if and only if each neighbourhood of a closed set F contains closure of some neighbourhood of F

14. Prove that every metric space is normal.

15. Show that a sub space of a completely regular space in completely regular.

16. Show that a topological space is connected if any only if Ø and X are the only subsets of X which are both open and closed.
17. Show that the space of all irrational numbers considered as a subspace of real line in totally disconnected.
18. Show that closure of any connected space is connected is connected
19. Show that a discrete space having only one point is connected and nay discrete space having more than one point is disconnected.
20. Define a Topology T on R such that (R, T) is disconnected.

Departemnt of MATHEMATICS,OU
Proposed Choice Based Credit System (CBCS)
M.Sc Mathematics

Semester -III

S.No.	Code No	Paper	Paper Title	Hrs/Week	Internal Assessment	Semester Exam	Total Marks	Credits
1. Core	MM 301	I	Complex Analysis	4	20	80	100	4
2. Core	MM 302	II	Functional Analysis	4	20	80	100	4
Elective	MM 303 A MM 303 B MM 303 C	III	Discrete Mathematics Analytic Number Theory Differential Geometry	4	20	80	100	4
Elective	MM 304 A MM 304 B MM 304 C	IV	Operation Research Numerical Techniques Algebric Number Theory	4	20	80	100	4
5. Practical	MM 351	Practical	Complex Analysis	4	….	50	50	2
6. Practical	MM 352	Practical	Functional Analysis	4	….	50	50	2
7. Practical	MM 353 A MM 353 B MM 353 C	Practical	Discrete Mathematics Analytic Number Theory Differential Geometry	4	….	50	50	2
8. Practical	MM 354 A MM 354 B MM 354 C	Practical	Operation Research Numerical Techniques Algebric Number Theory	4	….	50	50	2
			Total :	**32**				**24**

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. Mathematics

MM301 Semester IIIComplex Analysis
Paper-I

UNIT-I

Regions in the Complex Plane -Functions of a Complex Variable - Mappings -Mappings by the Exponential Function- Limits - Limits Involving the Point at Infinity - Continuity -Derivatives - Cauchy–Riemann Equations -Sufficient Conditions for Differentiability - Analytic Functions - Harmonic Functions -Uniquely Determined Analytic Functions - Reflection Principle - The Exponential Function -The Logarithmic Function -Some Identities Involving Logarithms -Complex Exponents -Trigonometric Functions -Hyperbolic Functions

UNIT-II

Derivatives of Functions w(t) -Definite Integrals of Functions w(t) - Contours -Contour Integrals - Some Examples -Examples with Branch Cuts -Upper Bounds for Moduli of Contour Integrals –Anti derivatives -Cauchy–Goursat Theorem -Simply Connected Domains- Multiply Connected Domains- Cauchy Integral Formula -An Extension of the Cauchy Integral Formula -Liouville's Theorem and the Fundamental Theorem of Algebra -Maximum Modulus Principle

UNIT-III

Convergence of Sequences - Convergence of Series - Taylor Series -Laurent Series -Absolute and Uniform Convergence of Power Series- Continuity of Sums of Power Series - Integration and Differentiation of Power Series - Uniqueness of Series Representations-Isolated Singular Points - Residues -Cauchy's Residue Theorem - Residue at Infinity - The Three Types of Isolated Singular Points - Residues at Poles -Examples -Zeros of Analytic Functions -Zeros and Poles -Behavior of Functions Near Isolated Singular Points

UNIT-IV

Evaluation of Improper Integrals -Improper Integrals from Fourier Analysis - Jordan's Lemma - Indented Paths - - Definite Integrals Involving Sines and Cosines - Argument Principle -Rouché's Theorem -Linear Transformations -The Transformation w = 1/z - Mappings by 1/z -Linear Fractional Transformations -An Implicit Form -Mappings of the Upper Half Plane

Text: James Ward Brown, Ruel V Churchill, Complex Variables with applications

Complex Analysis

MM351Paper-I Semester -III

Practical Questions

1

In each case, determine the singular points of the function and state why the function is analytic everywhere except at those points:

(a) $f(z) = \dfrac{2z+1}{z(z^2+1)}$; *(b)* $f(z) = \dfrac{z^3+i}{z^2-3z+2}$; *(c)* $f(z) = \dfrac{z^2+1}{(z+2)(z^2+2z+2)}$.

2

Show that $u(x, y)$ is harmonic in some domain and find a harmonic conjugate $v(x, y)$ when

(a) $u(x, y) = 2x(1-y)$; *(b)* $u(x, y) = 2x - x^3 + 3xy^2$;
(c) $u(x, y) = \sinh x \sin y$; *(d)* $u(x, y) = y/(x^2+y^2)$.

3

Find all values of z such that

(a) $e^z = -2$; *(b)* $e^z = 1 + \sqrt{3}i$; *(c)* $\exp(2z-1) = 1$.

4

Let the function $f(z) = u(x, y) + iv(x, y)$ be analytic in some domain D. State why the functions

$$U(x, y) = e^{u(x,y)} \cos v(x, y), \quad V(x, y) = e^{u(x,y)} \sin v(x, y)$$

are harmonic in D and why $V(x, y)$ is, in fact, a harmonic conjugate of $U(x, y)$.

5

Show that

(a) $(1+i)^i = \exp\left(-\dfrac{\pi}{4} + 2n\pi\right) \exp\left(i\dfrac{\ln 2}{2}\right) \quad (n = 0, \pm 1, \pm 2, \ldots)$;
(b) $(-1)^{1/\pi} = e^{(2n+1)i} \quad (n = 0, \pm 1, \pm 2, \ldots)$.

6

Let C denote the line segment from $z = i$ to $z = 1$. By observing that of all the points on that line segment, the midpoint is the closest to the origin, show that

$$\left| \int_C \frac{dz}{z^4} \right| \le 4\sqrt{2}$$

without evaluating the integral.

7

Show that if C is the boundary of the triangle with vertices at the points 0, $3i$, and -4, oriented in the counterclockwise direction (see Fig. 48), then

$$\left| \int_C (e^z - \overline{z})\, dz \right| \le 60.$$

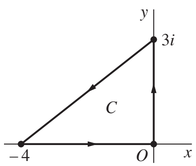


FIGURE 48

8

Let C be the unit circle $z = e^{i\theta}(-\pi \le \theta \le \pi)$. First show that for any real constant a,

$$\int_C \frac{e^{az}}{z}\, dz = 2\pi i.$$

Then write this integral in terms of θ to derive the integration formula

$$\int_0^{\pi} e^{a\cos\theta} \cos(a \sin\theta)\, d\theta = \pi.$$

9

Find the value of the integral of $g(z)$ around the circle $|z - i| = 2$ in the positive sense when

(a) $g(z) = \dfrac{1}{z^2+4}$; *(b)* $g(z) = \dfrac{1}{(z^2+4)^2}$.

10

Show that for R sufficiently large, the polynomial $P(z)$ in Theorem 2, Sec. 53, satisfies the inequality

$$|P(z)| < 2|a_n||z|^n \quad \text{whenever} \quad |z| \ge R.$$

11

Obtain the Maclaurin series representation

$$z\cosh(z^2) = \sum_{n=0}^{\infty} \frac{z^{4n+1}}{(2n)!} \qquad (|z| < \infty).$$

12

Obtain the Taylor series

$$e^z = e\sum_{n=0}^{\infty} \frac{(z-1)^n}{n!} \qquad (|z-1| < \infty)$$

for the function $f(z) = e^z$ by

13

In each case, show that any singular point of the function is a pole. Determine the order m of each pole, and find the corresponding residue B.

(a) $\dfrac{z^2+2}{z-1}$; *(b)* $\left(\dfrac{z}{2z+1}\right)^3$; *(c)* $\dfrac{\exp z}{z^2+\pi^2}$.

14

Evaluate the integral

$$\int_C \frac{\cosh \pi z}{z(z^2+1)}\, dz$$

when C is the circle $|z| = 2$, described in the positive sense.

15

Show that

(a) $\operatorname*{Res}_{z=\pi i} \dfrac{z - \sinh z}{z^2 \sinh z} = \dfrac{i}{\pi}$;

(b) $\operatorname*{Res}_{z=\pi i} \dfrac{\exp(zt)}{\sinh z} + \operatorname*{Res}_{z=-\pi i} \dfrac{\exp(zt)}{\sinh z} = -2\cos(\pi t)$

16

Evaluate

$$\int_{-\infty}^{\infty} \frac{\cos x\, dx}{(x^2+a^2)(x^2+b^2)} \quad (a > b > 0).$$

17

Derive the integration formula

$$\int_0^\infty \frac{\cos(ax) - \cos(bx)}{x^2}\,dx = \frac{\pi}{2}(b-a) \qquad (a \geq 0, b \geq 0).$$

Then, with the aid of the trigonometric identity $1 - \cos(2x) = 2\sin^2 x$, point out how it follows that

$$\int_0^\infty \frac{\sin^2 x}{x^2}\,dx = \frac{\pi}{2}.$$

18

Evaluate

$$\int_0^\pi \frac{\cos 2\theta\, d\theta}{1 - 2a\cos\theta + a^2} \quad (-1 < a < 1)$$

19

Suppose that a function f is analytic inside and on a positively oriented simple closed contour C and that it has no zeros on C. Show that if f has n zeros z_k $(k = 1, 2, \ldots, n)$ inside C, where each z_k is of multiplicity m_k, then

$$\int_C \frac{zf'(z)}{f(z)}\,dz = 2\pi i \sum_{k=1}^{n} m_k z_k.$$

20

Determine the number of zeros, counting multiplicities, of the polynomial

(a) $z^4 + 3z^3 + 6$; *(b)* $z^4 - 2z^3 + 9z^2 + z - 1$; *(c)* $z^5 + 3z^3 + z^2 + 1$

inside the circle $|z| = 2$.

Department of Mathematics
Osmania University
M.Sc Mathematics

MM302 Semester-III

Functional Analysis
Paper-II

Unit –I

NORMED LINEAR SPACES: Definitions and Elementary Properties, Subspace, Closed Subspace, Finite Dimensional Normed LinearSpaces and Subspaces, Quotient Spaces, Completion of Normed Spaces.

Unit-II

HILBERT SPACES: Inner Product Space, Hilbert Space, Cauchy-Bunyakovsky-Schwartz Inequality, Parallelogram Law, Orthogonality, Orthogonal Projection Theorem, Orthogonal Complements, Direct Sum, Complete Orthonormal System, Isomorphism between Separable HilbertSpaces.

Unit-III

LINEAR OPERATORS: Linear Operators in Normed Linear Spaces, Linear Functionals, The Space of Bounded Linear Operators, Uniform Boundedness Principle,Hahn-Banach Theorem, Hahn-Banach Theorem for Complex Vector and Normed Linear Space, The General Form of Linear Functionals in Hilbert Spaces.

Unit-IV

FUNDAMENTAL THEOREMS FOR BANACH SPACES AND ADJOINT OPERATORS IN HILBERT SPACES: Closed Graph Theorem, Open Mapping Theorem, Bounded Inverse Theorem, Adjoint Operators, Self-Adjoint Operators, Quadratic Form, Unitary Operators, Projection Operators.

Text Book:

A First Course in Functional Analysis-Rabindranath Sen, Anthem Press An imprint of Wimbledon Publishing Company.

Reference:

1. Introductory Functional Analysis- E.Kreyzig- John Wilely and sons, New York,
2. Functional Analysis, by B.V. Limaye 2nd Edition.
3. Introduction to Topology and Modern Analysis- G.F.Simmons. Mc.Graw-Hill International Edition.

M.Sc. Mathematics
Practicals Questions

MM 352

Semester-III

Functional Analysis

Paper-II

1. Let ρ be the matric induce by a norm on a linear space $E \neq \phi$. If ρ_1 is defined by

$$\rho_1(x, y) = \begin{cases} 0 & x = y \\ 1 + \rho(x, y) & x \neq y \end{cases}$$

then prove that ρ_1 can't be obtain from a norm on E.

2. (a). Show that the closure $\bar{X}$ of a subspace X of a normed linear space E is again a subspace of E. (b). Prove that the intersection of an arbitrary collection of non-empty closed subspaces of the normed linear space E is a closed subspace of E.

3. Let E_1 be a closed subspace and E_2 be a finite dimensional subspace of a normed linear space E. Then show that $E_1 + E_2$ is closed in E.

4. Show that a finite dimensional normed linear space is separable.

5. Show that equivalent norms on a vector space E induces the same topology on E.

6. Let C be a convex set in a Hilbert space H, and $d = inf\{\|x\|, x \in C\}$.If $\{x_n\}$ is a sequence in C such that $\lim_{x\to\infty}\|x_n\| = d$, show that $\{x_n\}$ is a Cauchy sequence.

7. Show that if M and N are closed subspaces of a Hilbert space H, then $M + N$ is closed provided $x \perp y$ for all $x \in M$ and $y \in N$.

8. Let $\{a_1, a_2, ...a_n\}$ be an orthogonal set in a Hilbert space H, and $\alpha_1, \alpha_2, ...\alpha_n$ be scalars such that their absolute values are respectively 1. Show that $\|\alpha_1 a_1 + ... + \alpha_n a_n\| = \|a_1 + a_2 + ... + a_n\|$.

9. Let H be a Hilbert space, $M \subseteq H$ a convex subset, and x_n a sequence in M such that $\|x_n\| \to d$ as $n \to \infty$ where $d = \inf_{x \in M} \|x\|$.Show that $\{x_n\}$ converges in H.

10. Let $x_1, x_2, ...x_n$ satisfy $x_i \neq 0$ and $x_i \perp x_j$ if $i \neq j$, $i, j = 1, 2, ...n$. Show that the $x_i's$ are linearly independent and extend the Pythagorean theorem from 2 to n dimensions.

11. Let E be a linear space over a scalar field $\mathbb{R}(or\mathbb{C})$. Prove that the space of continuous linear operators mapping E into itself is a ring.

12. Prove that every linear operator on a normed space is continuous iff bounded.

13. Give an example of an linear operator which is not bounded. Explain.

14. Let $x_0 \neq 0$ be a fixed element in a normed linear space E. Then prove that there exists a linear functional $f(x)$, defined on the entire space E, such that $\|f\| = 1$ and $f(x_0) = \|x_0\|$.

15. Let L be a closed linear subspace of a normed linear space E, and x_0 be a vector not in L. If d is the distance from x_0 to L, show that there exists a functional $f_0 \in E^*$ such that $f_0(L) = 0$, $f_0(x_0) = 1$ and $\|f_0\| = \frac{1}{d}$.

16. Given that E is a Banach space, $\mathcal{D}(T) \subseteq E$ is closed, and the linear operator T is bounded, show that T is closed.

17. Prove that for the projections P_1 and P_2 to be orthogonal, it is necessary and sufficient that the corresponding subspace L_1 and L_2 are orthogonal.

18. Give an example of a normal operator which is neither self-adjoint nor unitary. Explain.

19. Let $\|.\|$ and $\|.\|'$ be norms on a linear space E. Then prove that the norm $\|.\|$ is stronger than $\|.\|'$ if and only if there is some $\alpha > 0$ such that $\|x\| \leq \alpha \|x\|'$ for all $x \in E$.

20. Prove that P is a self-adjoint operator with its norm equal to one and P satisfies $P^2 = P$.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY

M.Sc. Mathematics

MM –303 A | Semester -III

Discrete Mathematics

Paper-III (A)

UNIT- I

LATTICES: Partial Ordering – Lattices as Posets – some properties of Lattices – Lattices as Algebraic Systems – Sublattices, Direct products and Homomorphisms – some special Lattices – Complete, complemented and distributive lattices.
(Pages 183-192, 378-397 of [1])

UNIT- II

BOOLEAN ALGEBRA: Boolean Algebras as Lattices – Bolean Identities – the switching Algebra – sub algebra, Direct product and homomorphism – Join irreducible elements – Atoms (minterms) – Boolean forms and their equivalence – minterm Boolean forms – Sum of products canonical forms – values of Boolean expressions and Boolean functions – Minimization of Boolean functions – the Karnaugh map method.
(Pages 397 – 436 of [1])

UNIT- III

GRAPHS AND PLANAR GRAPHS **:** Directed and undirected graphs – Isomorphism of graphs – subgraph – complete graph – multigraphs and weighted graphs – paths – simple and elementary paths – circuits – connectedness – shortest paths in weighted graphs – Eulerian paths and circuits – Incoming degree and outgoing degree of a vertex - Hamiltonian paths and circuits – Planar graphs – Euler's formula for planar graphs.
(Pages 137-159, 168-186 of [2])

UNIT- IV

TREES AND CUT-SETS: Properties of trees – Equivalent definitions of trees - Rooted trees – Binary trees – path lengths in rooted trees – Prefix codes – Binary search trees – Spanning trees and Cut-sets – Minimum spanning trees
(Pages 187-213 of [2])

Text Books:-

[1] J P Tremblay and R. Manohar: Discrete Mathematical Structures with applications to Computer Science, McGraw Hill Book Company
[2] C L Liu : Elements of Discrete Mathematics, Tata McGraw Hill Publishing Company Ltd. New Delhi. (Second Edition).

M.Sc. Mathematics
Practicals Questions

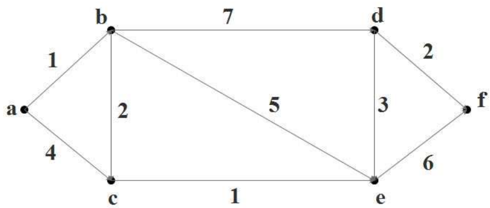
MM 353A

Semester-III

Discrete Mathematics

Paper-III (A)

1. Show that in a lattice if $a \leq b$ and $c \leq d$, then $a * c \leq b * d$.
2. Show that every interval of a lattice is a sublattice.
3. Prove that every chain is a distributive lattice.
4. Prove that the direct product of any two distributive lattice is a distributive lattice.
5. Show that Demorgan's laws hold in a complemented distributive lattice.
6. Write the boolean expression $x_1 \oplus x_2$ in an equivalent sum-of-products canonical form and product-of-sum canonical form in three variables x_1, x_2 and x_3.
7. Show that the following boolean expression are equivalent to one another (a) $(x \oplus y) * (x' \oplus z) * (y \oplus z)$.
(b) $(x * z) \oplus (x' * y) \oplus (y * z)$.
(c) $(x \oplus y) * (x' \oplus z)$.
(d) $(x * z) \oplus (x' * y)$.
8. Obtain simplified boolean expression which is equivalent to the expression $m_6 + m_7 + m_9 + m_{11} + m_{13}$.
9. Use the Karnaugh map representation to find a minimal sum-of-products expression of the function $f(a, b, c, d) = \sum(0, 1, 2, 3, 13, 15)$.
10. Expand the function $f(w, x, y, z) = w + \overline{y}z + \overline{x}y$ into their canonical sum-of-product form.
11. List all non-isomorphic simple graphs on 4 or fewer vertices.
12. Apply Dijkstras algorithm to determine a shortest path between a and f in the following weighted graph.



13. Give an example of a graph which has both an Eulerian circuit and Hamilton circuit.

14. Show that a complete directed graph always possesses a Hamiltonian.

15. Let G be a connected planar graph with ν vertices e edges where $\nu \geq 3$ then $e \leq 3\nu - 6$.

16. Built a binary search tree for a given sequence of number $17, 23, 4, 7, 9, 19, 45, 6, 2, 37, 99$.

17. How many different (pairwise non-isomorphic) trees are there of order 6 that is with 6 vertices.

18. Prove that an undirected graph with $e = \nu - 1$ that has no circuit is a tree.

19. Prove that every circuit has an even number of edges in common with every cut-set in a connected graph.

20. Prove that a tree with two or more vertices have at least two leaves.

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. Mathematics

MM – 303 B **Semester -III**

Analytic Number Theory

Paper III B

Unit I

Averages of arithmetical function: The big oh notation- Asymptotic equality of functions- Euler summation formula- Some asymptotic formulas- The average order of d(n)- The average order of the divisor functions σα(n)- The average order of Ø(n)- An application to the distribution of lattice points visible trona the origin- The average order of μ(n) andΛ(n)- The partial sums of dirichlet product- Applications to μ(n) and Λ(n)- Another identity for the partial sums of a dirichlet product. (Sections3.1 to 3.12)

Unit II

Some elementary theorems on the distribution of prime numbers- Introduction chebyshev's functions- ψ(x) and θ(x)- Relation connecting θ(n)and π(n)- Some equivalent forms of the prime number theorem- Inequalities for π(n) and p_n. (Sections4.1 to 4.5)

Unit III

Shapiro's Tauberian theorem- Applications of shapiro's theorem An asymptotic formula for the partial sums $\sum_{p\leq x} 1/p$ - The partial sums of the mobins function- Selberg Asymptotic formula. (Sections4.6 to 4.11 except 4.10)

Unit-IV

Finite Abelian groups and their character: Construction of sub groups- Characters of finite abelian group- The character group- The orthogonality relations for characters Dirichlet characters- Sums involving dirichelt characters the non vanishing of L(1,χ) for real non principal χ . (Sections 6.4 to 6.10)

Text Books:

[1] Tom M. Apostol- Introduction to Analytic Number Theory.

M.Sc. Mathematics

Analytic Number Theory

MM 353 B Paper III B Semester III

Practical Questions

1. Use Euler's summation formula to deduce

$$\sum_{n\le x}\frac{\log n}{n}=\frac{1}{2}\log^2 x+A+0(\frac{\log x}{x})\text{ where A is a constant}$$

2. If x ≥1 Prove that

$$\sum_{n\le x}\phi(n)=\frac{1}{2}\sum_{n\le x}\mu(n)\left[\frac{x}{n}\right]^2+\frac{1}{2}$$

3. Show that density of lattice points visible from the origin in $6/\pi^2$
4. Show that $\sum_{n\le x}d(n)=x\log x+O(x)$. Where d(n) = $\sum_{d/n}1$
5. Fluid average order of $\underset{\alpha}{\sigma}(n)$ for all $\alpha\varepsilon R$
6. Find the highest power of 10 that divides 1000 !
7. Prove that for every n >1 there exists n consecutive composite numbers
8. Let $f(x)=x^2+x+41$.Find the smallest integer x≥ 0 for which f(x) is composite
9. Define Chebyshev's functions $\chi(n)$ and $\xi(n)$ and show that

$$\psi(x)=\sum_{m\le\log_2^x}\theta\left(x^{\frac{1}{m}}\right)$$

10. Show that $\psi(x)=\sum_{m\le\log_2^x}\sum_{p\le x^{\frac{1}{m}}}\log p$
11. Showthat for all $x\ge1$ we have

$$\sum_{n\le x}\Psi(\frac{x}{n})=x\log x-x+o(\log x)$$

12. Show that for all $x\ge1$ we have $\sum_{n\le x}\theta(\frac{x}{n})=x\log x+0(x)$

13. Show that the following are equivalent

1. $\pi(x)=\frac{x}{\log x}+0(\frac{x}{\log_x^2})$

2. $\theta(x)=x+0(\frac{x}{\log x})$

14. Let f be an arithmetic function such that

$$\sum_{p\le x} f(p)\log p=(ax+b)\log x+cx+0(1)\, for\, x\ge 2$$

Then prove that there is a constant A depending on f such that if $x\ge 2$

$$\sum_{p\le x} f(p)=ax+(a+c)(\frac{x}{\log x}+\int_2^x \frac{dt}{\log^2 t})+b\log(\log x)+A+0(\frac{1}{\log x})$$

15. Show that $\lim_{x\to\infty}\frac{\mu(x)}{x}=\lim_{x\to\infty}\frac{H(x)}{x\log x}$

16. Let Gbe a finite group and G^1 be a Sub group of G For any element $a\in G$ define indicator of a in G. Suppose $a\in G$ and $a\notin G^1$. Construct a subgroup of G containing both $a\ and\ G^1$

17. Prove that the set of all Characters in a finite abelian group from an abelian group w.r.t to multiplication

18. State and Prove orthogonality relations DirchletcharactorsmoduloK

19. Find the Dirichlet characters for K=3,4,5

20. Let G be a set of n^{th} roots of a non-zero complex number. If G is a group under multiplication prove that G is a group of n^{th} roots of unity.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. Mathematics

MM –303(C) Semester III

Differential Geometry

Paper-III(C)

Unit I

Space Curves, Tangent Line, Contact of order of a curve and a surface, Osculating Plane, Principal normal, Binormal, Torsion-Curvature-Serret-Frenet formulae-Examples thereon, The Osculating Circle-Osculating Sphere-Helices Involutes and Evolutes-Examples thereon.

Unit II

CurvesonSurfaces tangent plane-Normal, Parametric curves, First order magnitudes-Second order magnitudes-Direction coefficients-Double family of curves, Curvature of normal section-Meunier's theorem-Examples thereon.

Unit III

Principal directions and curvatures-First curvatures Gaussian curvatures, Euler's theorem. The surface z=f(x,y), Surface of revolution-Examples thereon, Geodesics, Normal property of Geodesics-Geodesics curvature, Torsion-Joachimsthal's theorem.

Unit IV

Envelops characteristics-Edge of regression-Developable surfaces-Osculating developable-Polar developable-Rectifying developable,Envelopes-Characteristic points-Examples thereon.

Text Book:

[1] C.E. Weatherburn, Differential Geometry of three dimensions, (E.L.B.S.Edition,1964).

Reference Books:

[2] T.J. Willmore, An Introduction to differential geometry(Oxford University press), 11th Edition, New Delhi,1993.

[3] Mittal and Agarwal, Differential Geometry(Krishna Prakashan Media (P) Ltd.) 12th Edition.

M.Sc. (Mathematics)

MM353(c) Differential Geometry Semester –III

Paper –III(C)

Practical Questions

(1) Define contact of nth order of a curve and a surface. Prove that if the circle $lx+my+nz=0$, $x^2+y^2+z^2=cz$ has three pt contact with paraboloid $ax^2+by^2=2z$ then $C=\frac{(l^2+m^2)}{(bl^2+cm^2)}$

(2) State and prove Serret-Frenet formulae. Prove that $[\mathbf{t}^{I},\mathbf{t}^{II},\mathbf{t}^{III}]=k^5\left(\frac{\tau}{k}\right)'$

(3) For the given curve $\mathbf{r}=(e^{-u}\sin u, e^{-u}\cos u, e^{-u})$. Find at any point of this curve(i) unit tangent vector **t**, (ii) the equation of tangent(iii). The equation of normal plane, (iv) the unit principal normal **n**, (v) the curvature, (vi) the equation of principal normal, (vii) the unit binormal vector **b**, (viii) the equation of binormal.

(4) Find the centre and radius of Osculating Sphere. Prove that the curve given by $x=a\sin u$, $y=0$, $z=a\cos u$ lies on a sphere.

(5) Show that a necessary and sufficient condition for a curve be helix is that the ratio of the curvature and torsian is constant. Find the equation involute and find its curvature

(6) Define two fundamental forms. Determine the unit-normal and the fundamental forms of the surfaces.

$r=(u\cos v, u\sin v, f(v))$

(7) Calculate the fundamental magnitudes for the Monge's form of the surface $z=f(x,y)$. Calculate the fundamental magnitudes and normal to the surface $2z=ax^2+2hxy+by^2$ taking x,y as parameters.

(8) Define Direction coefficients and ratios. Find the condition that the two families represented by the equation $Pdu^2+2Qdudv+Rdv^2=0$ are orthogonal if and only if $ER-2FQ+GP=0$ by using tangent to these directions.

(9) Show that the curves $du^2-(u^2+a^2)dv^2=0$ form an orthogonal system on the right helicoid $\mathbf{r}=(u\cos v, u\sin v, av)$ and show that on a right helicoid, the family of curves orthogonal to the curves $u\cos v=\text{constant}$ is the family $(u^2+a^2)\sin^2 v=\text{constant}$.

(10) Define normal curvature . Show that the normal to the surface

$x=(u+v)/\sqrt{2}$, $y=(u-v)/\sqrt{2}$, $z=uv$ at a point (u,v) is
$\mathbf{n}=(x,-y,-1)/\sqrt{(1+x^2+y^2)}=(u+v,\ v-u,-\sqrt{2})/(\sqrt{2})\sqrt{(1+u^2+v^2)}$also evaluate curvature of normal section.

(11) Define principal directions and principal curvatures and Derive the equations.

(12) Show that the principal radii of curvature of the surface $y\cos(z/a)=x\sin(z/a)$ are equal to $\pm(x^2+y^2+a^2)/a$ and find principal directions(Lines of Curvature).

(13) Define Gaussian curvature and find the Gaussian curvature at any point of the right helicoid $\mathbf{r}=(u\cos v,\ u\sin v,\ av)$. Hence show that a right helicoid is a minimal surface.

(14) Find the Geodesics on a surface of Revolution $(u\cos v,\ u\sin v,\ f(u))$.

(15) Determine curvature and torsion of geodesics. If k, τ are curvature and torsion of a Geodesic, Prove that $\tau^2=(k-k_a)(k_b-k)$.

(16) Define envelope and write its equation.If a plane makes intercepts a,b,c on the axes, so that $1/a^2+1/b^2+1/c^2=1/k^2$, show that its envelopes is a conicoid which has the axes as equal conjugate diameters.

(17) Define edge of regression and find equation of edge of regression of the envelope. A sphere of constant radius b have their centres on the fixed circle $x^2+y^2=a^2$, $z=0$. Prove that their envelope is the surface

$(x^2+y^2+z^2+a^2-b^2)=4a^2(x^2+y^2)$.

(18) Find the condition that $z=f(x,y)$ may represents a developable surface. Prove that surface $xy=(z-c)^2$ is developable.

(19) Prove that curve itself is the edge of regression of the osculating developable and also prove that the edge of regression of envelope of the normal planes is the locus of the centre of osculating spheres.

(20) Find the equation of the developable surface which passes through the curves

$z=0$, $y^2=4ax$, $x=0$, $y^2=4bz$

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Mathematics)

MM – 304A **Semester III**

Operations Research
Paper IV A

Unit I
Formulation of Linear Programming problems, Graphical solution of Linear Programming problem, General formulation of Linear Programming problems, Standard and Matrix forms of Linear Programming problems, Simplex Method, Two-phase method, Big-M method, Method to resolve degeneracy in Linear Programming problem, Alternative optimal solutions. Solution of simultaneous equations by simplex Method, Inverse of a Matrix by simplex Method, Concept of Duality in Linear Programming, Comparison of solutions of the Dual and its primal.

Unit II
Mathematical formulation of Assignment problem, Reduction theorem, Hungarian Assignment Method, Travelling salesman problem, Formulation of Travelling Salesman problem as an Assignment problem, Solution procedure.
Mathematical formulation of Transportation problem, Tabular representation, Methods to find initial basic feasible solution, North West corner rule, Lowest cost entry method, Vogel's approximation methods, Optimality test, Method of finding optimal solution, Degeneracy in transportation problem, Method to resolve degeneracy, Unbalanced transportation problem.

Unit III
Concept of Dynamic programming, Bellman's principle of optimality, characteristics of Dynamic programming problem, Backward and Forward recursive approach, Minimum path problem, Single Additive constraint and Multiplicatively separable return, Single Additive constraint and Additively separable return, Single Multiplicatively constraint and Additively separable return.

Unit-IV
Historical development of CPM/PERT Techniques - Basic steps - Network diagram representation - Rules for drawing networks - Forward pass and Backward pass computations - Determination of floats - Determination of critical path - Project evaluation and review techniques updating.

Text Books:
[1] S. D. Sharma, Operations Research.
[2] Kanti Swarup, P. K. Gupta and Manmohan, Operations Research.
[3] H. A. Taha, Operations Research – An Introduction.

M.Sc.(Mathematics)
Operation Research
MM 354A Paper IV A Semester III
Practical Questions

1. Find a geometrical interpretation and solution as well for the following LPP
Maximize $z=3x_1+ 5x_2$ subject to restrictions: $x_1+ 2x_2\leq 2000$, $x_1+ x_2\leq 1500$, $x_2\leq 600$ and $x_1\geq 0$, $x_2\geq 0$.
2. Solve the following LPP geometrically
Max. $z=8000\, x_1+7000x_2$, subject to $3x_1+ x_2\leq 66$, $x_1+ x_2\leq 45$, $x_1\leq 20$, $x_2\leq 40$ and $x_1\geq 0$, $x_2\geq 0$.
3. Using Simplex method, solve the following LPP
Min. $z= x_1-3x_2+2x_3$ subject to $3x_1-x_2+3x_3\leq 7$, $-2x_1+4x_2\leq 12$, $-4x_1+3x_2+8x_3\leq 10$, and x_1, x_2, $x_3\geq 0$.
4. Use two-phase simplex method to solve the problem
Min. $z= x_1-2x_2-3x_3$ subject to the constraints $-2x_1+x_2+3x_3=2$, $2x_1+3x_2+4x_3=1$, and x_1, x_2, $x_3\geq 0$.
5. Solve by using Big-M Method for the problem
Max. $z=-2x_1-x_2$, subject to $3x_1+x_2=3$, $4x_1+3x_2\geq 6$, $x_1+2x_2\leq 4$ and $x_1\geq 0$, $x_2\geq 0$.
6. A department head has four subordinates, and four tasks to be performed. Subordinates differ in efficiency and tasks in their intrinsic difficulty. Time each man would take to perform each task is given in effectiveness matrix. How the tasks should be allocated to each person so as to minimize the total man-hour.

		Subordinates			
		I	II	III	IV
Tasks	A	8	26	17	11
	B	13	28	4	26
	C	38	19	18	15
	D	19	26	24	10

7. There are 5 jobs to be assigned on 5 machines and associated cost matrix is as follows:

		Machines				
		I	II	III	IV	V
Jobs	A	11	17	8	16	20
	B	9	7	12	6	15
	C	13	16	15	12	16
	D	21	24	17	28	26
	E	14	10	12	11	15

Find the optimum assignment and associated cost using assignment technique.

8. Given the matrix of set-up costs, show how to sequence the production so as to minimize the set up cost per cycle.

	A_1	A_2	A_3	A_4	A_5
A_1	∞	2	5	7	1
A_2	6	∞	3	8	2
A_3	8	7	∞	4	7
A_4	12	4	6	∞	5
A_5	1	3	2	8	∞

9. Consider the following transport problem

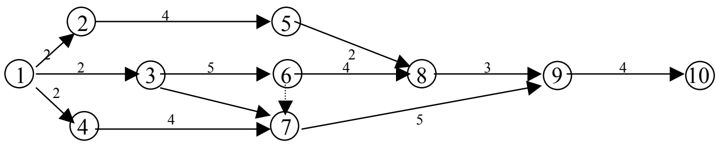
Source	Destination				Total
	D_1	D_2	D_3	D_4	
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
Total	20	40	30	10	100

Determine the initial feasible solution.

10. Find the initial basic feasible solution of the following transport problem by North West Corner Method.

		Warehouse				Factory Capacity
		W_1	W_2	W_3	W_4	
Factory	F_1	19	30	50	10	7
	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
Warehouse Requirement		5	8	7	14	34

11. Find the value of max $(y_1y_2y_3)$ subject to $y_1+y_2+y_3\geq 5$ and $y_1, y_2, y_3\geq 0$.
12. Minimize $z=y_1^2+ y_2^2+ y_3^2$ subject to $y_1+y_2+y_3\geq 15$ and $y_1, y_2, y_3\geq 0$.
13. Use dynamic programming to show that $-\sum_{i=1}^{n} p_i log p_i$, subject to $\sum_{i=1}^{n} p_i = 1$ is maximum, when $p_1=p_2=...=p_n=1/n$.
14. Use the principle of optimality to find the maximum value of $z=b_1x_1+b_2x_2+...+b_nx_n$ where $x_1+ x_2+...+ x_n=c$, and $x_1, x_2,..., x_n\geq 0$, $b_1, b_2,..., b_n>0$.
15. Solve the following problem using dynamic programming: Minimize $z= y_1^2+ y_2^2+...+y_n^2$ subject to the constraints $y_1= y_2=...= y_n=b$ and $y_1, y_2,..., y_n\geq 0$.
16. Consider a project given by the following Network diagram. Numbers along various activities represent the normal time of completion of that activity D_{ij}



Find minimum time of completion of the project.

17. A project has the following time schedule (in months)

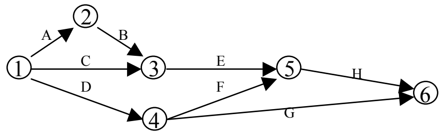
Activity	1→2	1→3	1→4	2→5	3→6	3→7	4→6	5→8	6→9	7→8	8→9
Normal Duration	2	2	1	4	8	5	3	1	5	4	3

Draw the network diagram to represent the above project. Find the critical path also.

18. A project has the following time schedule. Draw the Network diagram to represent this project. Find the critical path also.

Activity	1→2	1→3	1→4	2→5	3→6	3→7	4→7	5→8	6→8	7→9	8→9	9→10
Normal Duration	2	2	2	4	5	8	4	2	4	5	3	4

19. A project is represented by following Network diagram.



The time estimate of activities are as given below.

Activity	A	B	C	D	E	F	G	H
t_0	4	5	8	2	4	6	8	3
t_p	8	10	12	7	10	15	16	7
t_m	5	7	11	3	7	9	12	5

Determine the minimum expected time of completion of the project. Also determine the critical path.

20. A project has the following time schedule

Activity	Time in weeks	Activity	Time in weeks
1 →2	4	5 →7	8
1 →3	1	6 →8	1
2 →4	1	7 →8	2
3 →4	1	8 →9	1
3 →5	6	8 →10	8
4 →9	5	9 →10	7
5 →6	4		

Construct a PERT network and compute

(i) T_E and T_L for each event

(ii) Float for each activity and

(iii) critical path and its duration.

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. (Mathematics)

MM – 304 B Semester

IIINumerical Techniques

Paper IVB

Unit I

Transcendental and polynomial equations: Introduction, Bisection method, Iteration methods based on first degree equation; Secant method, Regulafalsi method, Newton-Raphson method, Iteration method based on second degree equation; Mullers method, Chebyshev method, Multipoint iterative method, Rate of convergence of secant method, Newton Raphson method,(Algorithms of above methods)

Unit II

System of linear algebraic equation: Direct methods, Triangularization method, Cholesky method, Partition method, Iteration method: Jacobi Iteration method Gauss seidel Iterative method, SOR method.

Unit III

Interpolation and Approximation: Introduction,Lagrange and Newton's divided difference interpolation,Finite difference operators, Hermiteinterpolation,piecewise and Spline Interpolation,least square approximation.(Algorithms on Lagrange and Newton divided difference Interpolation).

Unit IV

Numerical Integration: methods based on Interpolation, Newton's cotes methods, methods based on Undertermined coefficients, Guasslegendre Integration method, Numerical methods ODE: Singlestep methods: Eulers method, Taylor series method, Rungekutte second and forth order methods, Multistep methods: Adam Bash forth method, Adam Moulton methods, Milne-Simpson method predictor and corrector methods. (Algorithms on Trapezoidal, Simpson, Eulers&Runggekutte. methods only)

Text Book:

[1] Numerical Methods for Scientific and Engineering computation by M.K. Jain, S.R.K. Iyengar, R.K. Jain, New Age Int. Ltd., New Delhi.

[2] Computer Oriented Numerical Methods by V. Rajaraman.

Reference:

[1] Introduction to Numerical Analysis, by S.S. SastryPrentice Hall Flied.

M.Sc. (Mathematics)

MM354B Numerical Techniques **Semester –III**

Paper –IVB

Practical Questions

1. Use the bisection method to find the solution accurate to within 10^{-5} for the problem $2x\cos(2x)-(x+1)^2=0 \ for \ -3\le x\le 2$
2. Use the Newton-Raphson method to find the solution accurate to within 10^{-5} for the problem $\sin x-e^x=0 \ for \ 0\le x\le 1,\ 3\le x\le 4,\ 6\le x\le 7$
3. The fourth degree polynomial $f(x)=230x^4+18x^3+9x^2-22x$ has two real zeros, one in [-1,0] and the other in [0,1]. Attempt to approximate these zeros to within 10^{-6} by using (a) Method of False position (b) Secant method (c)Mullers Method
4. The fourth degree polynomial $f(x)=230x^4+18x^3+9x^2-22x$ has two real zeros, one in [-1,0] and the other in [0,1]. Attempt to approximate these zeros to within 10^{-6} by usingChebeshev method
5. The fourth degree polynomial $f(x)=230x^4+18x^3+9x^2-22x$ has two real zeros, one in [-1,0] and the other in [0,1]. Attempt to approximate these zeros to within 10^{-6} by using Multipoint iterative method
6. Use LU decomposition method to solve the system of equations $2x_1-x_2=1,\ -x_1+2x_2-x_3=0,\ -x_2+2x_3-x_4=0,\ -x_3+2x_4=1$
7. Use Cholesky method to find the factorization of the matrix $A=LL'$ for the matrix
$$\begin{bmatrix} 4 & -1 & 1 \\ -1 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$
8. Explain Gauss- seidelinterative method to solve the system of equations Ax=b
9. Use Gauss-Seidel method to solve the system of equations $x_1+2x_2-2x_3=-1,\ x_1+x_2+x_3=2,\ 2x_1+2x_2+x_3=5,$
10. The linear system Ax=b given by $4x_1+3x_2=24,\ 3x_1+4x_2-x_3=30,\ -x_2+4x_3=-24,$ has the solution $(3,4,-5)^T$ compare the iteration from the Gauss-Seidel method and the SOR method with w=1.25 using $x^{(0)}=(1,1,1)^T$ for the methods.
11. Use appropriate lagrange interpolating polynomials degree one, two and three to approximate $f(0.43)$ if $f(0.1)=1,\ f(0.25)=1.64872,\ f(0.5)=2.71828,\ f(0.75)=4048169$
12. Derive Newton's divided difference interpolating polynomial.
13. Compute the divided difference polynomial for the data

x_i	1.0	1.3	1.6	1.9	2.2
$f(x_i)$	0.7651977	0.6200860	0.4554022	0.2818156	0.1103623

14. Derive Hermiteinterpolateory polynomial.

15. Use the Hermite polynomial that agrees with the data listed in table to find approximation of $f(1.5)$,

x_i	$f(x_i)$	$f'(x_i)$
1.3	0.6200860	-0.5220232
1.6	0.4554022	-0.5698959
1.9	0.2818186	-0.5811571

16. Apply Taylor's method of order two and four with N=10 to the initial value problem $y' = y - t^2 + 1,\ 0 \le t \le 2,\ y(0) = 0.5$
17. Derive Runge-Kutta method of order Two.
18. Use the Runge-Kutta method of order 4 with h=0.2, N=10 and $t_i = 0.2$; to obtain approximation to the solution of the initial value problem $y' = y - t^2 + 1,\ 0 \le t \le 2,\ y(0) = 0.5$
19. Derive Adams-Moulton method of order 4.
20. Derive Adams-Bashforth method of order 4.

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. Mathematics

MM – 304C Semester III

Algebraic Number Theory

Paper IVC

Unit I

The Gaussian integers – Introduction – The Fundamental theorem of arithmetic in the Gaussian integers - The two square problems.

Unit II

Arithmetic in quadratic fields – Introduction - Quadratic fields - The integers of a quadratic field - Binary quadratic forms - Modules

Unit III

The coefficient ring of a module - The Unit theorem - Factorization theory in quadratic field - The failure of unique factorization.

Unit IV

Generalized congruences and norm of a module - Product and Sum of modules - The Fundamental factorization theorem.

Text Book:

[1] William W. Adam, Lory Joel Goldstein, Introduction to Number Theory.

(Sections: 7.1 to 7.3, 8.1 to 8.7, 9.1 to 9.4)

M.Sc. Mathematics

Algebraic Number Theory

MM 354C Paper IV C Semester-III

Practical Questions

1. Use the Division Algorithm for Gaussian Integers to determine the quotient and remainder when β is divided by α where

 i) $\alpha = 5 - 2i, \quad \beta = 6 + i$
 ii) $\alpha = 3 + 15i, \quad \beta = 187 + 46i$
2. Let α and β be Gaussian Integers. If $\alpha|\beta$ show that $N(\alpha)|N(\beta)$.
 Does the converse hold? Justify.
3. Find gcd of
 i. $5 + i,\ 2 - i$
 ii. $2 + 4i,\ 6 - 2i$
4. By factorizing rational primes construct
 i. a table of primes of norm ≤ 50.
 ii. a table of primes of norm ≤ 100.
5. Suppose $p_1, p_{2,\ldots,} p_r$ are distinct primes such that $p_i \equiv 1(mod 4)$ for $i = 1,2, \ldots, r$. Show that the number of solutions of the Diophantine equation $x^2 + y^2 = p_1, p_{2,\ldots,} p_r$ is 4^r. Hence find the solutions of
 i. $x^2 + y^2 = 20$
 ii. $x^2 + y^2 = 65$
 iii. $x^2 + y^2 = 32045$
6. Show that α, β in $Q(\sqrt{\alpha})$ are linearly independent if and only if $\begin{vmatrix} \alpha & \alpha' \\ \beta & \beta' \end{vmatrix} \neq 0$.
 Hence verify whether the following are linearly independent or not. If not exhibit a linear dependent relation between them
 i. $1, \sqrt{5}$ ii. $\frac{1}{2}, 2 - 3\sqrt{2}$ iii. $1, \frac{1}{2}$ iv. $2, 2 - 3\sqrt{-2}$ v. $1, a + b\sqrt{3}$ where $a, b \in Q$
7. Suppose d is a square free integer such that $d \equiv 1(mod 4)$.
 Show that S_d consists of all numbers of the form $\frac{x + y\sqrt{d}}{2}$ where x and y are rational integers of the same parity(i,e both even or both odd).
8. Find the discriminant and associated module of the following quadratic forms
 i. $x^2 + xy + y^2$ ii. $2x^2 + 3xy - y^2$
9. i. Find D_M for $M = \{1, \sqrt{3}\}$, $M = \{2 + \sqrt{3}, 5 - 2\sqrt{3}\}$ and $M = \{\frac{\sqrt{-5}}{3}, \frac{1}{2} - \frac{2}{5}\sqrt{-5}\}$
 ii. Suppose α, β and α_1, β_1 belong to $Q(\sqrt{d})$ and assume that $\alpha_1 = r\alpha + s\beta$ and $\beta_1 = t\alpha + u\beta$ for r, s, t, u rational. Show that $D(\alpha_1, \beta_1) = (ru - st, D(\alpha, \beta))$
10. Show that $\{1, \sqrt{3}\} = \{5 + 17\sqrt{3}, 2 + 7\sqrt{3}\}$. Compute the discriminant of these two bases and verify that they are equal.

11. Find θ_M for the following modules.

i. $M = \{1, \sqrt{3}\}$ ii. $M = \{2 + \sqrt{3}, 5 - 2\sqrt{3}\}$ iii. $M = \{7, 4\sqrt{2}\}$ iv. $M = \{1, Lw_d\}$ where L is a positive rational integer.

V. $M = \{a, b\sqrt{\alpha}\}$ where a, b, α are pair wise relatively prime rational integers.

12. Suppose M is a module such that $\alpha M = M$ for all α in I_d. Show that $\theta_M = I_d$

13. Compute the fundamental unit of I_d when

i. $d = 3$ ii. $d = 6$ iii. $d = 17$ iv $d = 35$

14. Compute the fundamental unit of $\theta_L = \{1, Lw_d\}$ if

i. $L = 4, d = 3$ ii. $L = 2, d = 3$ iii. $L = 5, d = 5$ iv. $L = 7, d = 5$

15. Determine the co-efficient ring of the module associated with the forms

i. $x^2 - 2xy + y^2$ ii. $x^2 + 3xy - 2y^2$

16. Find $N(M)$ for

i. $M = \{2 + 2\sqrt{3}, 6 + 2\sqrt{3}\}$ ii. $M = \{\frac{1}{2}, 2 + \sqrt{3}\}$ iii. $M = \{2, \sqrt{3}\}$ iv. $M = 11,3 + 2\sqrt{2}\}$

17. Let $M \subseteq A$ be modules. Let $n = (A: M)$

Show that $nA \subseteq M$

18. Find M_1M_2 and M_1+M_2 when

i. $M_1 = \{2, \sqrt{5}\}$, $M_2 = \{3, 1 + \sqrt{5}\}$

ii. $M_1 = \{1 - \sqrt{-5}, 1 + \sqrt{-5}\}$, $M_2 = \{12, 3 + \sqrt{-5}\}$

iii. $M_1 = \{2, 3\sqrt{7}\}$, $M_2 = \{3, 6\sqrt{7}\}$

iv. $M_1 = \{2, 21\sqrt{5}\}$, $M_2 = \{6, 7\sqrt{5}\}$

19. Verify that $MM' = N(M)\theta_M$ for $M = \{5, 2 + \sqrt{-1}\}$

20. Show that $\{1, \sqrt{5}\}\{3, \sqrt{5}\} = \{1, \sqrt{5}\}$ since $\{1, \sqrt{5}\}$ is a co-efficient ring. Does this violate the condition $M\theta_M = \theta_M M = M$ for any module. Justify.

Departemnt of MATHEMATICS,OU

Proposed Choice Based Credit System (CBCS)

M.Sc Mathematics

Semester -IV

S.No.	Code No	Paper	Paper Title	Hrs/Week	Internal Assessment	Semester Exam	Total Marks	Credits
1. Core	MM 401	I	Advanced Complex Analysis	4	20	80	100	4
2. Core	MM 402	II	General Measure Theory	4	20	80	100	4
3.Elective	MM 403 A MM 403 B MM 403 C	III	Integral equations and Calculus of Varaiations Mechanics Finite Difference Method	4	20	80	100	4
4. Elective	MM 404 A MM 404 B MM 404 C	IV	Elementary Opearator Theory Prime Number Theory Advanced Opeartion Research	4 OR	20	80	100	4 OR
4. Elective	MM 404 D	IV	Project	6	....	….	150	6
5. Practical	MM 451	Practical	Advanced Complex Analysis	4	….	50	50	2
6. Practical	MM 452	Practical	General Measure Theory	4	….	50	50	2
7. Practical	MM 453 A MM 453 B MM 453 C	Practical	Integral equations and Calculus of Varaiations Mechanics Finite Difference Method	4	….	50	50	2
8. Practical	MM 454 A MM 454 B MM 454 C	Practical	Elementary Opearator Theory Prime Number Theory Advanced Opeartion Research	4	….	50	50	2
			Total :	**32**				**24**
9.Seminar			Seminar	2	…..	…..	25	1

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. Mathematics

MM401 Semester IV

Advanced Complex Analysis

Paper-I

UNIT-I

Entire Functions: Jensen's formula -Functions of finite order- Infinite products Generalities -Example: the product formula for the sine function -Weierstrass infinite products - Hadamard's factorization theorem

UNIT-II

The Gamma and Zeta Functions: The gamma function –Analytic continuation-Further properties of Γ -The zeta function -Functional equation and analytic continuation

UNIT-III

The Zeta Function and Prime Number Theorem: Zeros of the zeta function - Estimates for $1/\zeta(s)$ - Reduction to the functions ψ and ψ_1 -Proof of the asymptotics for ψ_1 - Note on interchanging double sums

UNIT-IV

Conformal Mappings: Conformal equivalence and examples -The disc and upper half-plane - Further examples -The Dirichlet problem in a strip -The Schwarz lemma; automorphisms of the disc and upper half-plane-Automorphisms of the disc - Automorphisms of the upper half-plane

Text Book : Elias M Stein, Rami Shakarchi , C*omplex Analysis*

References: Lars V Ahlfors, *Complex Analysis*

R P Boas, *Entire Functions*

Lars V Ahlfors, *Conformal Invariants*

Advanced Complex Analysis

Paper-I

MM451 Semester IV

Practical Questions

1

Prove that if $|z| < 1$, then

$$(1+z)(1+z^2)(1+z^4)(1+z^8)\cdots = \prod_{k=0}^{\infty}(1+z^{2^k}) = \frac{1}{1-z}.$$

2

Find the Hadamard products for:

(a) $e^z - 1$;

(b) $\cos \pi z$.

3

Prove that for every z the product below converges, and

$$\cos(z/2)\cos(z/4)\cos(z/8)\cdots = \prod_{k=1}^{\infty}\cos(z/2^k) = \frac{\sin z}{z}.$$

4

Show that the equation $e^z - z = 0$ has infinitely many solutions in $\mathbb{C}$.

5

Prove Wallis's product formula

$$\frac{\pi}{2} = \frac{2\cdot 2}{1\cdot 3}\cdot\frac{4\cdot 4}{3\cdot 5}\cdots\frac{2m\cdot 2m}{(2m-1)\cdot(2m+1)}\cdots.$$

6

Prove that

$$\Gamma(s) = \lim_{n\to\infty}\frac{n^s n!}{s(s+1)\cdots(s+n)}$$

whenever $s \neq 0, -1, -2, \ldots$.

7

Show that Wallis's product formula can be written as

$$\sqrt{\frac{\pi}{2}} = \lim_{n\to\infty}\frac{2^{2n}(n!)^2}{(2n+1)!}(2n+1)^{1/2}.$$

8

Use the fact that $\Gamma(s)\Gamma(1-s) = \pi/\sin\pi s$ to prove that

$$|\Gamma(1/2+it)| = \sqrt{\frac{2\pi}{e^{\pi t}+e^{-\pi t}}}, \qquad \text{whenever } t \in \mathbb{R}.$$

9

The **Beta function** is defined for $\mathrm{Re}(\alpha) > 0$ and $\mathrm{Re}(\beta) > 0$ by

$$B(\alpha,\beta) = \int_0^1 (1-t)^{\alpha-1}t^{\beta-1}\,dt.$$

(a) Prove that $B(\alpha,\beta) = \dfrac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$.

(b) Show that $B(\alpha,\beta) = \displaystyle\int_0^\infty \frac{u^{\alpha-1}}{(1+u)^{\alpha+\beta}}\,du$.

10

Prove that for $\mathrm{Re}(s) > 1$,

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{e^x - 1}\, dx.$$

11

Prove as a consequence that one has

$$(\zeta(s))^2 = \sum_{n=1}^\infty \frac{d(n)}{n^s} \quad \text{and} \quad \zeta(s)\zeta(s-a) = \sum_{n=1}^\infty \frac{\sigma_a(n)}{n^s}$$

for $\mathrm{Re}(s) > 1$ and $\mathrm{Re}(s-a) > 1$, respectively. Here $d(n)$ equals the number of divisors of n, and $\sigma_a(n)$ is the sum of the a^{th} powers of divisors of n. In particular, one has $\sigma_0(n) = d(n)$.

12

Show that if $\{a_m\}$ and $\{b_k\}$ are two bounded sequences of complex numbers, then

$$\left(\sum_{m=1}^\infty \frac{a_m}{m^s}\right)\left(\sum_{k=1}^\infty \frac{b_k}{k^s}\right) = \sum_{n=1}^\infty \frac{c_n}{n^s} \quad \text{where } c_n = \textstyle\sum_{mk=n} a_m b_k.$$

The above series converge absolutely when $\mathrm{Re}(s) > 1$.

13

Prove that for $\mathrm{Re}(s) > 1$

$$\frac{1}{\zeta(s)} = \sum_{n=1}^\infty \frac{\mu(n)}{n^s},$$

where $\mu(n)$ is the **Möbius function** defined by

$$\mu(n) = \begin{cases} 1 & \text{if } n = 1\,, \\ (-1)^k & \text{if } n = p_1 \cdots p_k, \text{ and the } p_j \text{ are distinct primes}\,, \\ 0 & \text{otherwise}\,. \end{cases}$$

Note that $\mu(nm) = \mu(n)\mu(m)$ whenever n and m are relatively prime. [Hint: Use the Euler product formula for $\zeta(s)$.]

14

Show that

$$\sum_{k|n} \mu(k) = \begin{cases} 1 & \text{if } n = 1, \\ 0 & \text{otherwise.} \end{cases}$$

15

Prove that the Dirichlet series

$$\sum_{n=1}^\infty \frac{a_n}{n^s}$$

converges for $\mathrm{Re}(s) > 0$ and defines a holomorphic function in this half-plane.

16

Does there exist a holomorphic surjection from the unit disc to $\mathbb{C}$?

17

Prove that $f(z) = -\frac{1}{2}(z + 1/z)$ is a conformal map from the half-disc $\{z = x + iy : |z| < 1,\ y > 0\}$ to the upper half-plane.

18

Prove that the function u defined by

$$u(x, y) = \operatorname{Re}\left(\frac{i+z}{i-z}\right) \qquad \text{and} \qquad u(0, 1) = 0$$

is harmonic in the unit disc and vanishes on its boundary. Note that u is not bounded in $\mathbb{D}$.

19

Show that if $f : D(0, R) \to \mathbb{C}$ is holomorphic, with $|f(z)| \leq M$ for some $M > 0$, then

$$\left| \frac{f(z) - f(0)}{M^2 - \overline{f(0)}f(z)} \right| \leq \frac{|z|}{MR}.$$

20

Prove that if $f : \mathbb{D} \to \mathbb{D}$ is analytic and has two distinct fixed points, then f is the identity, that is, $f(z) = z$ for all $z \in \mathbb{D}$.

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. Mathematics

MM – 402 **Semester IV**

General Measure Theory

Paper- II

Unit I

Measure spaces- Measurable functions- Integration- General Convergence theorem.

Unit II

Signed measures- The Radon- Nikodym theorem.

Unit III

Outer measure and measurability- The Extension theorem- The Product measure.

Unit-IV

Inner measure- Extension by sets of measure zero- Caratheodory outer measure

Text Books:

[1] Real Analysis (Chapters 11, 12)

By H.L. Royden , Pearson Ediation.

M.Sc.(Mathematics)

General Measure Theory

Paper II

MM 452 Semester IV

Practical Questions

1. Let f be a non negative Lebesgue measurable function on $\mathbb{R}$. For each Lebesgue measurable set $E$of $\mathbb{R}$ define $\mu(E) = \int_E f$ the Lebesgue integral of $f$over E. Show that μ is a measure on the σ −algebra of Lebesgue measurable subsets of $\mathbb{R}$.

2. Let $\mathfrak{M}$ be a σ −algebra of subsets of a set X and $\mu: \mathfrak{M} \to [0, \infty)$ be finitely additive set function. Prove that μ is a measure if and onlyif whenever $\{A_n\}$ is an ascending sequence in $\mathfrak{M}$ then

$\mu(\bigcup_{k=1}^{\infty} A_k) = \lim_{k\to\infty} \mu(A_k)$.

3. Let $(X, \mathfrak{M}, \mu)$ be a measure space and $E_1 \Delta E_2$ denote the symmetric difference of two subsets E_1 and E_2 of X. That is $E_1 \Delta E_2 = [E_1 - E_2] \cup [E_2 - E_1]$. Show that

i. If E_1 and E_2 are measurable and $\mu(E_1 \Delta E_2) = 0$ then $\mu(E_1) = \mu(E_2)$

ii. show that if μ is complete, $E_1 \in \mathfrak{M}$ and $E_2 - E_1 \in \mathfrak{M}$. Then $E_2 \in \mathfrak{M}$ if $\mu(E_1 \Delta E_2) = 0$

4. Let $(X, \mathfrak{M}, \mu)$ be a measure space and $X_0 \in \mathfrak{M}$. Define $\mathfrak{M}_0$ to be the collection of sets in $\mathfrak{M}$ that are subsets of X_0 and μ_0 the restriction of μ to $\mathfrak{M}_0$. Show that $(X_0, \mathfrak{M}_0, \mu_0)$ is a measure space.

5. Suppose $(X, \mathfrak{M}, \mu)$ is not complete. Let E be a subset of a set of measure zero that does not belong to $\mathfrak{M}$. Let $f = 0$ on X and $g = \chi_E$. Show that $f = g$ a.e on X which f is measurable and g is not.

6. Let $f: R \to R$ be a function that is Lebesgue integrable over R. For a Lebesgue measurable set E define $\gamma(E) = \int_E f dm$. Prove that γ is a signed measure on the Lebesgue measurable space (R, L). Find a Hahn-decomposition of R with respect to this signed measure.

7. Let μ be a measure and μ_1 and μ_2 are mutually singular measures on a measurable space $(X, \mathfrak{M})$ for which $\mu = \mu_1 - \mu_2$. Show that $\mu_2 = 0$. Use this to establish uniqueness assertion of th Jordan decomposition theorem.

8. Show that if E is any measurable set then

i. $-\gamma^-(E) \le \gamma(E) \le \gamma^+(E)$

ii. $|\gamma(E)| \le |\gamma|(E)$

9. Show that if γ_1 and γ_2 are two finite signed measures then so is $\alpha\gamma_{1+\beta}\gamma_2$ where α and βar real numbers. Show that $|\alpha\gamma| = |\alpha||\gamma|$ and $|\gamma_1 + \gamma_2| \le |\gamma_1| + |\gamma_2|$

10. In the question 6 if E is a Lebesgue measurable set such that $0 < \gamma(E) < \infty$. Find a positive set C contained in E such that $\gamma(C) > 0$. Also find a Jordan decomposition of γ.

11.Let S be a collection of subsets of a set X and $\mu: S \to [0, \infty]$ be a set of function. Define $\mu^*(\emptyset) = 0$ and for $\subseteq X, \sigma \neq \emptyset$. Define $\mu^*(E) = \inf \sum_{k=1}^{\infty} \mu(E_k)$ where the infimum is taken over all countable collections $\{E_k\}$ of sets in S that cover E. Prove that the set function $\mu^*: P(X) \to [0, \infty]$ is an outer measure called the outer measure induced by μ.

12. Let $\mu^*: P(X) \to [0, \infty]$ be an outer measure. Let $A \subseteq X, \{E_k\}_{k=1}^{\infty}$be a disjoint countable collection of measurable sets and $E = \bigcup_{k=1}^{\infty} E_k$. Show that $\mu^*(A \cap E) = \sum_{k=1}^{\infty} \mu^*(A \cap E_k)$.

13. Show that any measure that is induced by an outer measure is complete.

14. Let X be a set, $S = \{\emptyset, X\}$ and define $\mu(\emptyset) = 0$ and $\mu(X) = 1$. Determine the outer measure μ^* induced by the set function $\mu: S \to [0, \infty)$ and the σ –algebra of measurable sets.

15. On the collection S of all subsets of R define the set function $\mu: S \to R$ by setting $\mu(A)$ to be the number of integers in A. Determine the outer measure μ^* induced by μ and the σ –algebra of measurable sets.

16. Let $\mathcal{A}$ be a σ –algebra on X and $\mathfrak{M}$ a collection of subsets of X which is closed under countable unions and which has the property that each subset of a set in $\mathfrak{M}$ is in $\mathfrak{M}$. Show that the collection

$\mathfrak{B} = \{B: B = A \Delta M, A \in \mathcal{A}, M \in \mathfrak{M}\}$ is a σ –algebra.

17. i. If $\mu(X) < \infty$ prove that $\mu_*(E) = \mu(X) - \mu^*(\bar{E})$ and

ii. if $\mathcal{A}$ is a σ –algebra then prove that

$\mu^*(E) = \inf\{\mu(A): E \subset A, A \in \mathcal{A}\}$ and

$\mu_*(E) = \sup\{\mu(A): A \subset E, A \in \mathcal{A}\}$

18. Suppose μ is a measure on an algebra $\mathcal{A}$ of subsets of X and E is any subset of X. If $\mathfrak{B}$ is the algebra generated by $\mathcal{A}$ and E and if $\overline{\mu}$ and $\underline{\mu}$ are extensions of μ to $\mathfrak{B}$ such that $\overline{\mu}(E) = \mu^*(E)$ and $\underline{\mu}(E) = \mu_*(E)$. Prove that $\overline{\mu}$ and $\underline{\mu}$ are measures on $\mathfrak{B}$.

19. Suppose $\rho = \{B = (A \cap E) \cup (A' \cap \bar{E}): A, A' \in \mathcal{A}\}$ where $\mathcal{A}$ and E are as in problem 18. Prove that ρ is an algebra of subsets of X containing $\mathcal{A}$ and E.

20. Suppose (X, ρ) is a metric space and μ^* be an outer measure on X with the property that $\mu^*(A \cup B) = \mu^*(A) + \mu^*(B)$ whenever $\rho(A, B) > 0$. Prove that every closed set is measurable with respect to μ^*.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Mathematics)

MM – 403A Paper IIIA Semester IV

Integral Equations & Calculus of Variations

Integral Equations:

Unit I

Volterra Integral Equations: Basic concepts - Relationship between Linear differential equations and Volterra Integral equations - Resolvent Kernel of Volterra Integral equation. Differentiation of some resolvent kernels - Solution of Integral equation by Resolvent Kernel - The method of successive approximations - Convolution type equations - Solution of Integro-differential equations with the aid of the Laplace Transformation – Volterra integral equation of the first kind - Euler integrals - Abel's problem - Abel's integral equation and its generalizations.

Unit II

Fredholm Integral Equations: Fredholm integral equations of the second kind – Fundamentals – The Method of Fredholm Determinants - Iterated Kernels constructing the Resolvent Kernel with the aid of Iterated Kernels - Integral equations with Degenerated Kernels. Hammerstein type equation - Characteristic numbers and Eigen functions and its properties.

Green's function: Construction of Green's function for ordinary differential equations - Special case of Green's function - Using Green's function in the solution of boundary value problem.

Calculus of Variations:

Unit III

The Method of Variations in Problems with fixed Boundaries:

Definitions of Functionals – Variation and Its properties - Euler's' equation - Fundamental Lemma of Calculus of Variation-The problem of minimum surface of revolution - Minimum Energy Problem Brachistochrone Problem - Variational problems involving Several functions - Functional dependent on higher order derivatives - Euler Poisson equation.

Unit IV

Functional dependent on the Functions of several independent variables - Euler's equations in two dependent variables - Variational problems in parametric form - Application of Calculus of Variation - Hamilton's principle - Lagrange's Equation, Hamilton’s equations.

Text Books:

[1] M. KRASNOV, A. KISELEV, G. MAKARENKO, Problems and Exercises in Integral Equations (1971)

[2] S. Swarup, Integral Equations, (2008)

[3] L.ELSGOLTS, Differential Equation and Calculus of Variations, MIR Publishers, MOSCOW

M.Sc.(Mathematics)

Integral Equations & Calculus of Variations

MM453A   Paper IIIA   Semester IV

Practical Questions

1. From an Integral equation corresponding to the differential equation $y''' + xy'' + (x^2 - x)y = xe^x + 1$; $y(0) = y'(0) = 1;\ y''(0) = 1$
2. Convert the differential equation $y''' + xy'' + (x^2 - x)y = xe^x + 1$; with initial conditions $y(0) = y'(0) = 1,\ y''(0) = 0$; into Volterra's Integral Equations.
3. Solve the Integral Equations $\varphi''(x) + \varphi(x) + \int Sinh(x-t)\varphi(t)dt + \int_0^x Cosh(x-t)\varphi'(t)dt = Coshx$; $\varphi(0) = \varphi'(0) = 0$.
4. Solve the Integral Equations $\int_0^x \frac{\varphi(t)dt}{(x-t)^\alpha} = x^n$; $0 < \alpha < 1$;
5. With the aid of Resolvent Kernel, find the solution of the Integral equation $$\varphi(x) = e^x \sin x + \int_0^x \frac{2+\cos x}{2+\cos t}\varphi(t)dt$$
6. Solve the Integral Equations $\phi(x) - \lambda\int_0^1 \arccos t.\phi(t)dt = \frac{1}{\sqrt{1-x^2}}$
7. Find the Characteristic numbers and Eigen function of the Integral Equations $$\phi(x) - \lambda\int_0^1 \left(45x^2\log t - 9t^2\log x\right)\phi(t)dt = 0$$
8. **Applications of Green's function :** Construct Green's function for the homogeneous boundary value problem $y^{iv}(x) = 0;\ y(0) = y'(0) = 0;\ y(1) = y'(1) = 0$.
9. **Applications of Green's function :** Solve the Boundary Value problem $y^{iv}(x) = 1;\ y(0) = y'(0) = y''(1) = y'''(0) = 0$.
10. **Applications of Green's function :** Solve the Boundary Value problem $y'' + y = x^2$; $y(0) = y(\pi/2) = 0$.
11. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{1+y'^2}}{y}dx$
12. Test for an extremum the functional $v[y(x)] = \int_0^1 \left(xy + y^2 - 2y^2y'\right)dx$; $y(0) = 1;\ y(1)) = 2$.

13. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} (16y^2 - y''^2 + x^2)dx$

14. Determine the extremals of the functional $v[y(x)] = \int_{-l}^{l} (\frac{\mu}{2} y''^2 + \rho y)dx$ that satisfies the boundary conditions $y(-l) = y'(-l) = y(l) = y'(l) = 0$

15. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} (2yz - 2y^2 + y'^2 - z'^2)dx$

16. Find the extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} \left[y^2 + (y')^2 + \frac{2y}{Coshx} \right] dx$

17. Write the ***Ostrgradsky*** equation for the functional $v[z(x,y)] = \iint_D \left[\left(\frac{\partial z}{\partial x} \right)^2 - \left(\frac{\partial z}{\partial y} \right)^2 \right] dxdy$

18. **Applications of Hamilton's and Lagrange's equations:** Derive the equation of a vibrations of a Rectilinear Bar.
19. **Applications of Hamilton's and Lagrange's equations:** A particle of mass **m** is moving vertically under the action of gravity and a resistance force numerically equal to **k** times the displacement **x** from an equilibrium position. Obtain the Hamilton's and Euler's equation.
20. Use Hamilton's principle to find the equations for the small vibrations of a flexible stretching string of length ***l*** and tension **T** fixed at end points.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Mathematics)

MM 403 B Semester IV

Mechanics
Paper III B

Unit I

Newton's Law of Motion: Historical Introduction, Rectilinear Motion: Uniform Acceleration Under a Constant Force, Forces that Depend on Position: The Concepts of Kinetic and Potential Energy, Dynamics of systems of Particles:- Introduction - Centre of Mass and Linear Momentum of a system- Angular momentum and Kinetic Energy of a system, Mechanics of Rigid bodies- Planar motion:- Centre of mass of Rigid body-some theorem of Static equilibrium of a Rigid body- Equilibrium in a uniform gravitational field.

Unit II

Rotation of a Rigid body about a fixed axis, Moment of Inertia:- calculation of moment of Inertia Perpendicular and Parallel axis theorem- Physical pendulum-A general theorem concerning Angular momentum-Laminar Motion of a Rigid body-Body rolling down an inclined plane (with and without slipping).

Unit III

Motion of Rigid bodies in three dimension-Angular momentum of Rigid body products of Inertia, Principles axes-Determination of principles axes- Rotational Kinetic Energy of Rigid body- Momentum of Inertia of a Rigid body about an arbitrary axis- The momental ellipsoid - Euler's equation of motion of a Rigid body.

Unit IV

Lagrange Mechanics:-Generalized Coordinates-Generalized forces-Lagrange's Equations and their applications-Generalized momentum-Ignorable coordinates-Hamilton's variational principle-Hamilton function-Hamilton's Equations- Problems-Theorems.

Text Book:

[1] G.R.Fowles, Analytical Mechanics, CBS Publishing, 1986.

M.Sc.(Mathematics)

Mechanics

MM 353B MM453B Paper III B Semester IV

Practical Questions

1. Discuss the motion of particle sliding down a smooth inclined plane at an angle θ to the horizontal.
2. Discuss the centre of mass of Solid homogeneous sphere of radius a.
3. Discuss the centre of mass of Hemispherical shell of radius a.
4. Discuss the centre of mass of Quadrant of uniform circular lamina of radius b.
5. Find the centre of mass of area bounded by a parabola $y=x^2/b$ and line $y=b$.
6. Point the moment of inertia of following:
   a. Rectangular lamina about a line passing through centre and normal to it,
   b. Rectangular parallelepiped,
   c. Circular wire and disk,
   d. Elliptic disk,
   e. Hollow sphere about a diameter, Solid sphere about a diameter.
7. Point the moment of inertia of a hollow sphere about diameter, its external and internal radii being a and b.
8. Find the moment of inertia of a uniform circular cylinder of length b and radius a about an axis through the centre and perpendicular to the central axis.
9. A circular hoop of radius a swing as a physical pendulum about a point on the circumference. Find the period of oscillation for small amplitude if the axis of rotation is (a) normal to the plane of the hoop and (b) in the plane of the hoop.
10. Find the acceleration of a uniform circular cylinder rolling down an inclined plane.
11. Find the direction of the principle axis in the plane of rectangular lamina of sides a and b at a corner.
12. Find the principle moments of inertia of a square plate about a corner.
13. Find the directions of principle axes for the above problem.
14. Find the inertia tensor for a square plate of side l and mass m in a coordinate system OXYZ where O is at corner and X and Y are along the two edges. Also find angular momentum and kinetic energy of rotation.
15. A thin uniform rectangular plate is of mass m and dimension 2a x a. Choose coordinate system OXYZ such that the plate lies in the XY plane with origin at the corner, the long dimension being along the X axis. Find the following:
    a. The moments and products of inertia,
    b. The moment of inertia about the diagonal through the origin,
    c. The angular momentum about the origin if the plate is spinning with angular rate w about the diagonal through the origin,
    d. The kinetic energy in part c.
16. Derive the governing equation for 1D damped harmonic oscillation.
17. Find the governing equation for single particle in central field.
18. Find the governing equation for a particle sliding on a movable inclined plane.
19. A mass suspended at the end of a light spring having spring constant k is set into vertical motion. Use the Lagrange equation to find the equation of motion.
20. Find the acceleration of a solid uniform sphere rolling down a perfectly rough fixed inclined plane.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY
M.Sc. (Mathematics)

MM 403C Semester IV

Finite Difference Methods

Paper III C

Unit I

Partial differential Equations – Introduction - Difference method - Routh Hurwitz criterion - Domain of Dependence of Hyperbolic Equations. (1.1 to 1.4)

Unit II

Difference methods for parabolic partial differential equations - Introduction – One space dimension - two space dimensions - Spherical and cylindrical coordinate System.(2.1 to 2.3, 2.5)

Unit III

Difference methods for Hyperbolic partial differential equations - Introduction - one space dimensions - two space dimensions - First order equations.(3.1 to 3.4)

Unit IV

Numerical methods for elliptic partial differential equations – Introduction - Difference methods for linear boundary value problems - General second order linear equation - Equation in polar coordinates.(4.1 to 4.4)

Text Book:

[1] M. K. Jain, S. R. K. Iyengar, R. K. Jain,
Computational Methods for Partial Differential Equations, Wiley Eastern Limited, New Age International Limited, New Delhi.

M.Sc.(Mathematics)

Finite Diference Method

MM 453C Paper III C Semester IV

Practical Questions

1. Classify the PDE $\frac{\partial^2 u}{\partial x^2} + 2x\frac{\partial^2 u}{\partial x \partial y} + (1-y^2)\frac{\partial^2 u}{\partial y^2} = 0$.
2. Classify the PDE $u_{tt}+4u_{tx}+4u_{xx}+2u_x-u_t=0$ and find its characteristics. Reduce the equation to its standard form.
3. Classify the PDE and the find the characteristics of $\frac{\partial^2 u}{\partial t^2} + (5+2x^2)\frac{\partial^2 u}{\partial x \partial t} + (1+x^2)(4+x^2)\frac{\partial^2 u}{\partial x^2} = 0$.
4. In which part of the (x, y) plane is the following equation elliptic $u_{xx}+4u_{xy}+(x^2+4y^2)u_{yy} = \sin xy$.
5. Classify and calculate the characteristics of $u_{xx}-t^2u_{tt}=0$.
6. Solve the heat conduction equation $u_t=u_{xx}$ subject to the initial and boundary conditions $u(x,0)=\sin\pi x$, $0\le x\le 1$, $u(0,t)=u(1,t)=0$ using
   a. The Schmidt method
   b. Crank-Nicolson method
7. Solve the heat conduction equation $u_t=u_{xx}$ subject to the initial and boundary conditions $u(x,0)=\sin\pi x$, $0\le x\le 1$, $u(0,t)=u(1,t)=0$, $t>0$ using
   a. Laasonen method
   b. Dufort-Frankel method
8. Use Crank-Nicolson method and central differences for the boundary conditions to solve the IVP $u_t=u_{xx}$, $u(x,0)=1$, $0\le x\le 1$, $u_t(0,t)=u(0,t)$, $u_t(1,t)= -u(1,t)$, $t>0$, with steplength $h=1/3$ and $\lambda=1/3$. Integrate upto 2 times levels.
9. Find the solution of 2D heat conduction equation $u_t=u_{xx}+u_{yy}$ subject to initial condition $u(x,y,0)=\sin\pi x \sin\pi y$, $0\le x,y\le 1$, and the boundary conditions $u=0$, on the boundaries, $t\ge 1$, using explicit method with $h=1/3$ and $\lambda=1/8$.
10. Find the solution of 2D heat conduction equation $u_t=u_{xx}+u_{yy}$ subject to initial condition $u(x,y,0)=\sin\pi x \sin\pi y$, $0\le x,y\le 1$, and the boundary conditions $u=0$, on the boundaries, $t\ge 0$, using the Peaceman-Rachford ADI method. Assume $h=1/4$, $\lambda=1/8$ and integrate for one time-step.
11. Find the solution of the IBVP $u_{tt}=u_{xx}$, $0\le x\le 1$, subject to the initial and boundary conditions $u(x,0)=\sin\pi x$, $0\le x\le 1$, $u_t(x,0)=0$, and the boundary condition $u(0,t)=u(1,t)=0$, $t>0$, using Explicit scheme
12. Find the solution of the IBVP $u_{tt}=u_{xx}$, $0\le x\le 1$, subject to the initial and boundary conditions $u(x,0)=\sin\pi x$, $0\le x\le 1$, $u_t(x,0)=0$, and the boundary condition $u(0,t)=u(1,t)=0$, $t>0$, using Implicit scheme
13. Solve the IBVP equation $u_{tt}=u_{xx}+u_{yy}$ subject to initial condition $u(x,y,0)=\sin\pi x \sin\pi y$, $u_t(x,y,0)=0$, $0\le x,y\le 1$, $u(x,y,t)=0$, on the boundary, $t\ge 0$, using the D'yakonov split form with $\theta=1/2$. Assume $h=1/3$ and $r=1/3$. Perform the integration for one time step.

14. Find the solution of $u_t+u_x=0$ subject to the initial condition

$$u(x,0) = \begin{cases} 0, & x<0 \\ x, & 0\le x\le 1 \\ 2-x, & 1\le x\le 2 \\ 0, & x>2 \end{cases}$$

using the Lax-Wendroff formula with $h=1/2$ and $r=1/2$. Compute upto 2 time step.

15. Solve $u_t+u_x=0$ subject to the initial condition $u(0,x)=\sin\pi x$, $0\le x\le 1$, using diffusion difference scheme.
16. Find the solution of $u_{xx}+u_{yy}=0$ in R subject to Dirichlet condition $u(x,y)=x-y$ on ∂R, where R is the region inside the triangle with vertices (0,0), (7,0), (0,7) and ∂R is its boundary. Assume the step length $h=2$.
17. Solve the equation $u_{xx}+u_{yy}= -10(x^2+y^2+10)$ over the square region with sides $x=0$, $y=0$, $x=3$, $y=3$ with $u=0$ on the boundary and mesh length equal to one.
18. Solve the mixed boundary value problem

$$\begin{aligned} &u_{xx}+u_{yy}=0, && 0\le x,y\le 1 \\ &u=2x, && 0\le x\le 1,\ y=0 \\ &u=2x-1, && 0\le x\le 1,\ y=1 \\ &u_x+u=2-y, && x=0, 0\le y\le 1 \\ &u=2-y, && x=1, 0\le y\le 1 \end{aligned}$$

The analytical solution is $u(x,y)=2x-y$. Use five point formula with $h=k=1/3$.

19. Solve the mixed boundary value problem

$$\begin{aligned} &u_{xx}+u_{yy}=0, && 0\le x^2+y^2\le 1,\ x\ge 0,\ y\ge 0 \\ &u=0, && x=0,\ y=0 \\ &u_n=x-y, && x^2+y^2=1 \end{aligned}$$

Use five point formula with $h=1/2$.

20. Solve the BVP $u_{rr}+(1/r)u_r+u_{zz}= -1$, $0\le r<1$, $-1<z<1$, and $u=0$ on the boundary using the five point scheme with $h=k=1/2$.

DEPARTMENT OF MATHEMATICS
OSMANIA UNIVERSITY

M.Sc. Mathematics

Semester-IV

MM – 404A

Elementary Operator Theory

Paper-IV (A)

Unit I

Spectral theory in finite dimensional normed spaces - Basic concepts of spectrum - Resolvent sets - Spectral properties of bounded linear operators - Further properties of resolvent and spectrum. (Sections 7.1, 7.5)

Unit II

Compact linear operators on normed spaces - Properties of compact linear operators - Spectral properties of compact linear operators on normed spaces - Operator equations involving compact linear operators. (Sections 8.1, 8.2, 8.3 and 8.5 of [1])

Unit III

Spectral properties of bounded self adjoint linear operators - Further spectral properties of bounded linear operators - Positive operators - Square root of a positive operator. (Sections 9.1, 9.2, 9.3 and 9.4 of [1])

Unit IV

Projection operators - Properties of projection operators - Spectral family - Spectral family of a bounded self adjoint linear operator.
(Sections 9.5, 9.6, 9.7 and 9.8 of [1])

Text Book :

[1] E. Kreyszig : Introductory Functional Analysis, John Wiley and Sons, New York, 1978.

Reference Books:

[1] Brown and Page: Elements of Functional Analysis, D.V.N. Comp.
[2] B.V. Limaye : Functional Analysis, Wiley Eastern Limited, (2nd Edition)
[3] P.R.Halmos : A Hilbert Space Problem Book, D.Van Nostrand Company, Inc. 1967.

M.Sc. Mathematics
Practicals Questions

MM 454 A

Semester-IV

Elementary operator theory

Paper-IV (A)

1. Show that the eigenvalues of a Hermitian matrix $A = (\alpha_{jk})$ are real.

2. If a square matrix $A = (a_{jk})$ has eigenvalues $\lambda_j, j = 1, ..., n$, show that kA has the eigenvalues $k\lambda_j$ and $A^m (m \in N)$ has the eigenvalues λ_j^m.

3. Give an example of an operator with a spectral value which is not an eigen value.

4. Prove that Eigenvectors $x_1, ..., x_n$ corresponding to different eigenvalues $\lambda_1, ..., \lambda_n$ of a linear operator T on a vector space X constitute a linearly independent set.

5. If S, $T \in B(X, X)$, show that for any $\lambda \in \rho(S) \cap \rho(T)$,
$$R_\lambda(S) - R_\lambda(T) = R_\lambda(S)(T - S)R_\lambda(T).$$

6. Show that $T : l^2 \to l^2$ defined by $Tx = y = (\eta_j)$, $\eta_j = (\xi_j/2^j)$, is compact.

7. Prove that the range $R(T)$ of a compact linear operator T on a normed space X is separable.

8. Let T be a compact linear operator on a normed space X. Then for every $\lambda \neq 0$ the null space, $N(T_\lambda)$ of $T_\lambda = T - \lambda I$ is finite dimensional.

9. Let H be a Hilbert space, T be a bounded linear operator and T^* the Hilbert-adjoint operator of T. Show that T is compact if and only if T^*T is compact.

10. Let T be a compact linear operator on a normed space X and let $\lambda \neq 0$. Then $Tx - \lambda x = y$ has a solution x if and only if y is such that $f(y) = 0$ for all $f \in X'$ satisfying $T^\times f - \lambda f = 0$.

11. Prove that the spectrum $\sigma(T)$ of a bounded self-adjoint linear operator T on a complex Hilbert space H is real.

12. The residual spectrum $\sigma(T)$ of a bounded self-adjoint linear operator T on a complex Hilbert space H is empty.

13. Let T_1 and T_2 be bounded self-adjoint linear operators on a complex Hilbert space H and suppose that $T_1T_2 = T_2T_1$ and $T_2 \geqq 0$. Show that then $T_1^2T_2$ is self-adjoint and positive.

14. Let S and T be bounded self-adjoint linear operators on a Hilbert space H. If $S \geqq 0$, show that $TST \geqq 0$.

15. Let T and W be bounded linear operators on a complex Hilbert space H and $S = W^*TW$. Show that if T is self-adjoint and positive, so is S.

16. Show that a projection P on a Hilbert space H satisfies

$$0 \leqq P \leqq I$$

. Under what conditions will (i)$P = 0$, (ii) $P = I$?

17. If a sum $P_1 + ... + P_k$ of projections $P_j : H \to H$ (H is a Hilbert space) is a projection, show that

$$\|P_1 x\|^2 + ... + \|P_k x\|^2 \leqq \|x\|^2.$$

18. Let T be a self-adjoint linear operator on the unitary space $H = \mathbb{C}^n$ then obtain a representation of T in terms of projections.

19. Obtain the representation of the self-adjoint linear operator T with eigenvalues $\lambda_1 < \lambda_2 < ... < \lambda_n$ on the n-dimensional Hilbert space H.

20. Show that the difference $P = P_2 - P_l$ of two projections on a Hilbert space H is a projection on H if and only if $P_1 \leqq P_2$.

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. Mathematics

MM – 404B Prime Number Theory Semester -IV

Paper –IV (B)

Unit – I

Dirichlet theorem as primes in an arithmetic progression – primes of the form 4n-1,4n+1 – Dirichlet of primes in arithmetic progression – Dirichlet series and Euler products – the half plane of convergence of a dirichlet series –functuion defined by a dirichlet series – Multiplication of dirichlet series –Eulers products –the half plane of convergence of a dirichlet series.

Unit –II

Analytic properties of dirichlet series – dirichlet series with non – negative coefficients –dirichlet series expresses as exponentials of dirichlet series –Mean value simule for dirichlet series – on integral principle for the coefficient of a dirichlet series and for the partial sum of a dirichlet series.

Unit – III

The Function $\zeta(s)\, and \quad L(s,x)$ - integral representation for the Hurwitz function – a contour integral representation for the Hurwitz zeta function – Analytic continuation of the Hurwitz zeta function equation for the Riemann zeta function and Hurwitz zeta function.

Unit –IV

Analytic proof of the prime number theorem – plan of the proof – two lemmas proving $[\Psi_1(x)]/X^2$ implies a prime number theorem – a contour integral representation for $[\Psi_1(x)]/X^2$ - upper bounds for $|\zeta(s)|$ and $|\zeta(s)|$ near the line $\sigma = 1$ - the non vanishing of $\zeta(s)$ on the line $\sigma = 1$ - inequalities for $|1\zeta(s)|$ and $\frac{|\zeta(s)|}{|\zeta(s)|}$ - completion of the proof of prime number theorem.

Scope as in chapter: 7,11,12,13 of (1)

Text Book :

1. Tom.M.Apostol, Introduction to Analytic Number theory, Springer International Student Edition

Prime Number Theory

MM 454 B | Paper IVB | Semester IV

Practical Questions

1. Show that there are infinitely many primes
2. Show that for x>1

$$\sum_{p\le x}\frac{\log p}{p}=\frac{1}{\phi(k)}\log x+\sum_{r=2}^{\phi(K)}\overline{\Psi}r(h)\sum_{p\le x}\frac{\psi_r(p)\log p}{p}+o(1)\,p\equiv h(\operatorname{mod}k)$$

3. For x>1 and $\chi\neq\psi_1$, Prove that

$$\sum_{p\le x}\frac{\chi(P)\log p}{p}=-L^1(1,\chi)\sum_{n\le x}\frac{\mu(n)\chi(n)}{n}+0(1)$$

4. For x>1 Prove that

$$L(1,\chi)\sum_{n\le x}\frac{\mu(n)\chi(n)}{n}=0(1)$$

5. For $x\ge 1$ Pr*ove that*

$$\sum_{p\le x}\frac{\log p}{p}=\frac{1-N(K)}{\phi(K)}\log X+0(1)$$

$$P\equiv 1(\operatorname{mod}k)$$

Where N (K) = number of non-principal characters

$$\chi \operatorname{mod} k \; such \;\; that \;\; L(1,\chi)=0$$

6. Prove that $\xi(s)\sum_{n=1}^{\infty}\frac{\mu(n)}{n^s}=1\, if\ \sigma>1$
7. Prove that $\sum_{n=1}^{\infty}\frac{\mu(n)f(n)}{n^s}=\frac{1}{F(s)}\, whereF(s)=\sum\frac{f(n)}{n^s}$

   Which converges absolutely for $\sigma>\sigma$ a.

8. Prove that $\sum_{n=1}^{\infty}\frac{\phi(n)}{n^s}=\frac{\xi(s-1)}{\xi(s)}\, if\ \sigma>2$
9. Prove that $\sum_{n=1}^{\infty}\frac{\lambda(n)}{n^s}=\frac{\xi(2s)}{\xi(s)}\, if\sigma>1$
10. Prove that $\xi(s)\xi(s-\alpha)=\sum_{n=1}^{\infty}\frac{\sigma_\alpha(n)}{n^s}\, if\sigma>\max(1,1+\operatorname{Re}(\alpha))$

11. Show that $\xi(s)=\prod_{p}\frac{1}{1-p^{-s}}\, if\ \sigma>1$

12. Show that $\dfrac{\xi(s-1)}{\xi(s)}=\pi_{p}\dfrac{1-p^{-s}}{1-p^{1-s}}$

13. Show that $\xi(s)\xi(s-\alpha)=\pi_{p}\dfrac{1}{(1-p^{-s})(1-p^{\alpha-s})} if \sigma>\max\left(1,1+\mathrm{Re}(\alpha)\right)$

14. Show that $\dfrac{\xi(2s)}{\xi(s)}=\prod_{p}\dfrac{1}{1+p^{-s}}\ if \sigma>1$

15. Show that $L(S,\chi)=\prod_{P}\dfrac{1}{1-\chi(P)P^{-S}}\quad if \sigma>1$

16. Show that for $\sigma>1$ $\quad \xi^{1}(S)=-\sum_{n=1}^{\infty}\dfrac{\log n}{n^{s}}$

17. Show that $\quad -\dfrac{\xi^{1}(s)}{\xi(s)}=\sum_{n=1}^{\infty}\dfrac{\wedge(n)}{n^{s}}$

18. Show that $\quad L(S,\chi)=K^{-S}\sum_{r=1}^{k}\chi(r)\xi(s,\frac{r}{k})$

19. Outline the proof of Prime number Theorem

20. Show that $\xi(1+it)\neq 0$ for every real t.

DEPARTMENT OF MATHEMATICS

OSMANIA UNIVERSITY

M.Sc. Mathematics

MM 404C | Paper IV(c) | Semester IV

Advanced Operation Research

UNIT I

Characteristics of Game theory –Minimax(Maxmin) criterion and optimal strategy- Saddle points- Solution of Games with saddle points- Rectangular Games without saddle points-Minimax(Maxmin) principle for Mixed strategy Games- Equivalence of Rectangular Game and Linear programming problem- Solution of (m x n) Games by Simplex method-Arithmetic method for (2 x 2) Games-concept of Dominance- Graphical method for (3 x 3)Games without saddle point

UNIT II

Inventory Problems: Analytical structure of inventory Problem, ABC analysis, EOQ Problems with and without shortage, with (a) Production is instantaneous (b) finite constant rate (c) shortage permitted random models where the demand follows uniform distribution.

UNIT III

Non-Linear programming-unconstrained problems of Maxima and Minima- constrained problems of Maxima and Minima-Constraints in the form of Equations – Lagrangian Method-Sufficient conditions for Max(Min) of Objective function with single equality constraint –With more than one equality constraints-Constraints in the form of Inequalities-Formulation of Non-Linear programming problems-General Nonlinear programming problem-Canonical form-Graphical Solution

Unit IV

Quadratic programming- Kuhn- Tucker Conditions- Non- negative constraints, General quadratic programming problem- Wolfe's modified simples method-Beales's Method- Simplex method for quadratic Programming.

Text Books:

[1] S.D. Sharma, Operations Research.

[2] Kanti swarup,P.K. Gupta and Manmohan, Operations Research.

[3] O.L. Mangasarian, Non-Linear Programming, McGraw Hill, New Delhi.

M.Sc. (Mathematics)

Advanced Operation Research

Paper –IV C

MM 454 C Practical Questions Semester - IV

1. Define
   - (i) Competitive games
   - (ii) Pure Strategies
   - (iii) Mixed Strategies
   - (iv) Two –person , Zero –sum game
   - (v) Payoff matrix
   - (vi) Saddle point

2. Solve the following game by linear by linear programming problem and solve it by simplex method

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix} \end{array}$$

3. Explain the principle and rules of dominance to reduce the size of payoff matrix and hence solve the following game: $\begin{bmatrix} 8 & 15 & -4 & -2 \\ 19 & 15 & 17 & 16 \\ 0 & 20 & 15 & 5 \end{bmatrix}$

4. Solve the following game using the graphical method $\begin{array}{cc} & \underset{-}{B} \\ \underset{-}{A} & \begin{bmatrix} -6 & 7 \\ 4 & -5 \\ -1 & -2 \\ -2 & 5 \\ 7 & 6 \end{bmatrix} \end{array}$

5. In the following 3X3 game, find optimal strategies and the value of the game.

B

		I	II	III
A	I	3	-2	4
	II	-1	4	2
	III	2	2	6

6. Explain clearly the various costs that are involved in inventory problems with suitable examples – How they are inter related?
7. Derive economic order quantity model for an inventory problem when shortages of costs are not allowed.
8. Derive an expression for economic production quantity with uniform rate of replacement with no shortages.
9. Show that for a system where demand is deterministic and is a constant R units per unit time and the production rate is infinite, it is never optimal in comparison to have any lost sales.
10. What is 'ABC' analysis in the problem of inventory control of an organization using a large number of items
11. A positive quantity b is to be divided into n parts in such a way that the product of n parts is to be a maximum. Use Lagrange's multiplier technique to obtain the optimal sub-division.

12. Solve the non 0p linear programming problem:

$$\text{Optimize } z = 4x_1^2 + 2x_2^2 + x_3^2$$

$$\text{STC : } X_1 + X_2 + X_3 = 15$$

$$2X_1 - X_2 + 2X_3 = 20$$

$$\text{and } x_1, x_2, x_3 \geq 0$$

13. State and prove Kuhn –Tucker necessary and sufficient conditions in Non- linear programming.
14. Solve graphically the following problems

$$\text{Max } Z = 2X_1 + 3X_2 \qquad \text{subject to } X_1^2 + X_2^2 \leq 20,$$

$$X_1 X_2 \leq 8, \text{ and } \quad X_1 X_2 \geq 0$$

Verify the Kuhn-Tucker conditions hold for the maxima you obtain.

15. What is a NLPP? How to formulate it? Also write its canonical form.
16. Derive Kuhn –Tucker necessary conditions for an optimal solution to a quadratic programming problem.
17. What is Quadratic programming? Outline a method of solving it?

18. Use Wolfe's method to solve the following problems:

$$\text{Min } Z = X_1^{2+}X_2^{2}+X_3^{2}$$

$$\text{STC: } X_{1+}X_{2+}\ 3X_3 = 2$$

$$5X_{1+}2X_{2+}\ X_3 = 5$$

$$X_{1,}X_{2,}\ X_3 \geq 0$$

19. Solve the following problem by Beale's Method:

$$\text{Max } Z = 2X_1+2X_2-2X_2^{2}$$

$$\text{STC: } \quad X_1+4X_2 \leq 4$$

$$X_1+X_2 \leq 2$$

$$\text{and } x_1,x_2 \geq 0$$

20. Apply Scuplex method for the following QPP:

$$\text{Max } Z = (2X_1+3X_2+2)\ (X_2-5)$$

$$\text{STC: } X_1+X_2 \leq 1$$

$$4X_1+X_2 \geq 2$$

$$\&\ X, X_2\ \geq 0$$

MOOCs [Massive Online Open Courses] Free Resources

E-Learning:

- NPTEL :nptel.ac.in [Core Subjects Certification]
- C++ INSTITUTE :cppinstitute.org [C++ Certification]
- ORACLEEDUCATION :education.oracle.com [Java, DBMS Certification]
- BIG DATA UNIVERSITY :bigdatauniversity.com [Big Data Certification]
- COURSERA :coursera.org [Core Subjects Certification]
- CODEACADEMY :codecademy.com [Coding Certification]
- KHANACADEMY :khanacademy.org [Core Subjects Certification]
- PIXAR IN A BOX :khanacademy.org/partner-content/pixar
- VIDEOLECTURES :videolectures.net
- YOUTUBEEDU :plus.google.com/+YouTubeEDU/posts
- DISNEY RESEARCH :disneyresearch.com
- ALISON :alison.com [Core Subjects Certification]
- INTERNET ARCHIVE :archive.org

Freeware:

- SCILAB : scilab.org [MatLab Equivalent]
- GEOGEBRA :geogebra.org [Software for Class Room Teaching]

Search Engine:

- WOLFRAM ALPHA :wolframalpha.com [Computing Engine]
- CITESEER :citeseerx.ist.psu.edu [Searching Research Articles]
- DOAJ :doaj.org [Open Access to Journals]