DEPARTMENT OF MATHEMATICS

B.A. / B.Sc.-III (Practical) Examination 2010-2011

Subject : MATHEMATICS (New Syllabus)

Paper : IV(b)

QUESTION BANK

Time : 3 hours **Marks : 50**

UNIT-I (FOURIER SERIES)

1) Prove that the fourier series of $f(x) = x + x^2$ for $-1 < x < 1$, is

$$f(x) = \frac{1}{3} + \frac{2}{\pi}\sum_1^\infty (-1)^n \left(\frac{2cosn\pi x}{n^2\pi} - \frac{sinn\pi x}{n}\right)$$

2) If $f(x)$=1 when $0 < x < 1$, $f(x)$=2 when $1 < x < 3, f(x)=\frac{3}{4}$ when $x = 0, 1$ & 3

& f$(x+3)$=f(x) $\forall\, x$. Show that f $(x) = \frac{5}{9} - \frac{1}{\pi}\sum_1^\infty \frac{2sin\frac{n\pi}{3}}{n}\, cosn\pi\big(2x-1\big)\ \forall\, x$.

3) Expand in a series of sine and cosines of multiples of x, for function given by $f(x) = \pi + x$,

When $-\pi < x < 0$; $f(x) = \pi - x$ when $0 < x < \pi$; What is the sum of the series for

$x = \pm\pi$ and $x = 0$?

4) Find a) fourier sine series and b) fourier cosine series which represents $f(x) = \pi - x$ in $0 < x < \pi$.

5) Show that the fourier series which converges to $f(x)$ in $-\pi \le x \le \pi$ where $f(x) = x + x^2$ when

$-\pi < x < \pi$ and $f(x) = \pi^2$ when $x = \pm\pi$ is $\frac{\pi^2}{3} + 4\sum_1^\infty (-1)^n \left(\frac{cosnx}{n^2} - \frac{sinnx}{2n}\right)$.Deduce

that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$.

6) Obtain fourier series whose sum is equal to $f(x)$ where $f(x) = 0$ when $-\pi \le x < -\frac{\pi}{2}$,

$f\left(-\frac{\pi}{2}\right) = -\frac{\pi}{4}$; $f(x) = x$ when $-\frac{\pi}{2} < x < \frac{\pi}{2}$; $f\left(\frac{\pi}{2}\right) = \frac{\pi}{3}$; $f(x) = 0$

when $\frac{\pi}{2} < x \le \pi$.

7) If $f(x) = cosx$ for $0 < x < \pi$ and $f(x) = -cosx$ for $-\pi < x < 0$; show that fourier series which converges to $f(x)$ is $\frac{\pi}{4}(\frac{2}{1.3} sin2x + \frac{4}{3.5} sin4x + \frac{6}{5.7} sin6x + ...)$

8) Find the fourier series which represents $|sinx|$ in $-\pi \leq x \leq \pi$.

9) Show that fourier series for the function e^x in the interval $-\pi \leq x \leq \pi$ is

$$\frac{e^{\pi} - e^{-\pi}}{\pi}\left(\frac{1}{2} + \sum_1^{\infty} \frac{(-1)^n}{n^2+1} \ (cosnx - nsinnx)\right)$$

10) Show that the fourier series in interval $-\pi \leq x \leq \pi$ for the function

$$coskx = \frac{sink\pi}{\pi}\left(\frac{1}{k} - \frac{2kcosx}{k^2-1^2} + \frac{2kcos2x}{k^2-2^2}\right),$$ k being non integer.

11) Show that the half range fourier cosine series for f(x) in [0,π] is $f(x) = \frac{\pi^2}{16} - 2\sum_1^{\infty} \frac{\cos(4n-2)x}{(4n-2)^2}$,

where $f(x) = \frac{\pi x}{4}$ $0 \leq x \leq \frac{\pi}{2}$ and $\frac{\pi}{4}(\pi - x)$ when $\frac{\pi}{2} < x \leq \pi.$

12) Show that the half range fourier cosine series for f(x) in $0 \leq x \leq 1$ *is*

$f(x) = \frac{1}{x^2}\{\sum_1^{\infty} \frac{cos2n\pi x}{n^2}\}$ where $f(x) = x^2 - x + \frac{1}{6}$

13) Show that the half range fourier sine series for f(x) in $0 \leq x \leq 1$ *is*

$f(x) = \frac{-1}{\pi}\left\{\sum_1^{\infty} \frac{sin2n\pi x}{n}\right\}$ where $f(x) = x - \frac{1}{2}.$

14) Find the fourier series on the interval $-\pi < x < \pi$ for

$f(x) = -\frac{\pi}{2}$, when $-\pi < x < 0$ $f(x) = \frac{\pi}{2}$, when $0 < x < \pi.$

15) If $f(x) = \frac{a}{4} - x$ when $0 \leq x \leq \frac{a}{2}$ and $f(x) = \frac{-3a}{4} + x$ when $\frac{a}{2} \leq x \leq a$,Show that the fourier cosine series of f(x) $\forall x$ in $0 \leq x \leq a$, $f(x) = \frac{2a}{\pi^2}\sum_1^{\infty} \frac{1}{(2n-1)^2} \frac{\cos(4n-2)\pi x}{a}.$

16) Find the fourier series for the function $f(x)$, where $f(x) = x + \frac{x^2}{4}$ in the interval $-\pi < x < \pi.$

17) Show that the half range fourier sine series for the function $f(x)$, where $f(x) = x(\pi - x)$ in (0,π) is $\frac{8}{\pi}\sum_1^{\infty} \frac{\sin(2n-1)x}{(2n-1)^3}.$

18) Find the fourier series expansion of $f(x) = \frac{n-x}{2}$ in $0 < x < 2\pi.$

19) Find the fourier series expansion of $f(x) = e^{-x}$ in $(0,2\pi).$

20) Show that the fourier series for the function f(x) defined by $f(x) = 0$ when $-3 < x < 0$;

$f(x) = 1$ when $0 < x < 3$ and $f(0) = \frac{1}{2}$ is $\frac{1}{2} + \frac{2}{\pi}\sum_1^\infty \frac{\sin(2n-1)\pi x}{3(2n-1)}$ in $-3 < x < 3$.

UNIT-II (LAPLACE AND INVERSE LAPLACE TRANSFORMATION)

21) Show that L $\{(5e^{2t} - 3)^2\}$= $\frac{25}{p-4} - \frac{30}{p-2} + \frac{9}{p}; p > 4.$

22) Show that L$\{t^2$cosat$\}$=$\frac{2p(p^2-3a^2)}{(p^2+a^2)^2}; p > 0.$

23) Prove that $\int_0^\infty t^3 e^{-t}\, sintdt = 0.$

24) Find $\mathrm{L}\,\{\mathrm{F(t)}\}$ and $\mathrm{L}\,\{\mathrm{F'(t)}\}$, for the function given by $\mathrm{F(t)} = 2t\ when\ 0 \le t \le 1$ and F(t)=t when t>1.

25) Show that L$\{\frac{cosat-cosbt}{t}\}$=$\frac{1}{2}\log(\frac{p^2+b^2}{p^2+a^2})$

26) If F(t)=$\begin{cases} 3t, & 0 < t < 2 \\ 6, & 2 < t < 4 \end{cases}$ find L{F(t)},where F(t) has period 4.

27) Compute L{F(t) ,if F(t)=$\begin{cases} sint, & 0 < t < \pi, \\ 0, & \pi < t < 2\pi \end{cases}$ where F(t) has period 2π.

28) Find L{Sin$\sqrt{t}$}.

29) Find L{erf$\sqrt{t}$}.

30) Show that L$\{\frac{COS\sqrt{t}}{\sqrt{t}}\}$=$\sqrt{\frac{\pi}{p}}\, e^{\frac{-1}{4p}}$.

31) Evaluate i) $\mathrm{L}^{-1}\{\frac{1}{(P^3+1)}\}$; ii) $\mathrm{L}^{-1}\{\frac{6}{(2P-3)} - \frac{3+4p}{3p^2-16} + \frac{8-6p}{16p^2+9}\}$.

32) Evaluate i) $\mathrm{L}^{-1}\{\frac{P-1}{(P+3)(P^2+2P+2)}\}$; ii) $\mathrm{L}^{-1}\{\frac{3P+7}{(P^2-2P-3)}\}$.

33) State second shifting property and hence evaluate $\mathrm{L}^{-1}\{\frac{(P+1)e^{-\pi p}}{(P^2+P+1)}\}$.

34) Evaluate (i) $\mathrm{L}^{-1}\{\frac{p}{(p^2+a^2)^2}\}$; ii) $\mathrm{L}^{-1}\{\frac{p}{(p^2-16)^2}\}$.

35) Evaluate (i) $\mathrm{L}^{-1}\left\{\frac{1}{p}\log\left(1 + \frac{1}{p^2}\right)\right\}$; ii) $\mathrm{L}^{-1}\left\{\frac{1}{p^3(p^2+1)}\right\}$.

36) Evaluate (i) $L^{-1}\left\{\frac{1}{(p^2+4)(p+1)^2}\right\}$; ii) $L^{-1}\left\{\frac{6p^2+22p+18}{p^3+6p^2+11p+6}\right\}$.

37) Evaluate (i) $L^{-1}\left\{\frac{3p^3-3p^2-40p+36}{(p^2-4)^2}\right\}$; ii) $L^{-1}\left\{\frac{5p^2-15p-11}{(p+1)(p-2)^2}\right\}$.

38) State Heavisides expansion formula. Using it find $L^{-1}\left\{\frac{p+5}{(p+1)(p^2+1)}\right\}$.

39) Define convolution of two functions and using convolution theorem evaluate $L^{-1}\left\{\frac{1}{(p-2)(p+2)^{\ 2}}\right\}$.

40) Define Beta function using convolution theorem prove that B(m,n)= $\frac{(gamma\ of\ m)(gamma\ of\ n)}{(\ gamma\ of m+n)}$ $m > 0, n > 0$.

UNIT-III(FOURIER TRANSFORMATION)

41) Find fourier cosine transform of $f(x) = \frac{1}{1+x^2}$ and hence find fourier sine transformation of $\frac{x}{1+x^2}$.

42) Find fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$.

43) Find fourier cosine transform of $f(x) = e^{-x^2}$.

44) Find inverse fourier transform of $\tilde{f}\ (p) = e^{-|p|y}$.

45) Find f(x) if $\widetilde{f_s}(p)$ =$p^n e^{-ap}$.

46) Find f(x) if $\widetilde{f_c}(p) = \begin{cases} \frac{1}{\sqrt{2\pi}}\left(a - \frac{p}{2}\right) & if\ ,\ p < 2a. \\ 0 & if \quad p \geq 2a. \end{cases}$

47) Use sine inversion formula to obtain f(x) if $\widetilde{f_s}(p) = \frac{p}{p^2+1}$.

48) If $f(x) = \begin{cases} x & ; \ 0 < x < 1 \\ 2 - x & ; \ 1 < x < 2 \\ 0 & ; \quad x > \ 2 \end{cases}$ find fourier cosine and sine transform of $f(x)$.

49) Find fourier cosine transform of $f(x) = x^{m-1}$.

50) Find fourier transform of $f(x)$ if $f(x) = x^2, when\ |x| < a\ and\ \ f(x) = 0\ when\ \ |x| > a$.

51) Find fourier transform of $f(x)$ if $f(x) = x, when\ |x| \le a\ and\ f(x) = 0\ when\ |x| > a$.

52) Find the complex fourier transform of $e^{-|x|}$.

53) Find the finite fourier sine transform and cosine transform of $f(x) = x$.

54) Find the finite fourier cosine transform of $f(x)$ if $f(x) = -\frac{cosk(\pi-x)}{ksink\pi}$ in $(0, \pi)$ and find $f(x)$

if $\widetilde{f_c}\ (p) = \frac{\cos(2p\frac{\pi}{3})}{(2p+1)^2}$ in 0<x<1.

55) Find the finite fourier sine transform and cosine transform of $f(x) = x^2; 0 < x < \pi$.

56) Find the finite fourier sine transform $f(x)$ if $f(x) = x; 0 \le x \le \frac{\pi}{2}$ and $f(x) = \pi - x; \frac{\pi}{2} \le x < \pi$.

57) Find the finite fourier sine transform of $f(x)$ if $f(x) = sinnx$.

58) Find the finite fourier sine transform of $x(\pi^2 - x^2)$ and $x(\pi - x)$.

59)If $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ then find fourier transform of f(x) and usig parseval's identity prove that $\int_0^\infty (\frac{sint}{t})^2\, dt = \frac{\pi}{2}$.

60) If $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ then find fourier transform of f(x) and usig parseval's identity prove that $\int_0^\infty (\frac{sinx-xcosx}{x^3})^2\, dx = \frac{\pi}{15}$.

UNIT-IV(APPLICATIONS OF LAPLACE TRANSFORMATION TO ORDINARY DIFFERIENTIAL EQUATION AND FOURIER TRANSFORMATION TO INITIAL AND BOUNDARY VALUE PROBLEMS)

61) Solve $(D^2 + 2D + 1)y = 3te^{-t},\ t > 0$, subject to conditions $y = 4,\ Dy = 2$ when $t = 0$.

62) Solve $(D^2 - 3D + 2)y = 1 - e^{2t}$, subject to conditions $y = 1,\ Dy = 0$ when $t = 0$.

63) Solve $(D^3 + 1)y = 1,\ t > 0$, subject to conditions $y = Dy = D^2y = 0$, when $t = 0$.

64) Solve $(D^2 + 9)y = 18t$, if $y(0) = 0;\ y\left(\frac{\pi}{2}\right) = 0$.

65) Solve $(D^2 + 2D)y = 0$, if $y(0) = 0;\ y(-1) = 1$.

66) Solve $(D^2 + 5D + 6)y = 5e^t$, subject to conditions $y = 2,\ Dy = 1$ when $t = 0$.

67) Solve $(D^3 - 3D^2 + 3D - 1)y = t^2e^t$; , subject to conditions $y = 1,\ Dy = 0\ , D^2\,y = -2$ when

$t = 0$.

68) Solve $ty'' + y' + 4ty = 0$; $y(0) = 3, y'(0) = 0$.

69) Solve $y'' - ty' + y = 1$; $y(0) = 1, y'(0) = 2$.

70) Solve $y'' + ty' - y = 0$; $y(0) = 0, y'(0) = 1$.

71) Solve $(D^2 - 1)x + 5Dy = t;\ -2Dx + (D^2 - 4)y = -2$; $if\ x = 0 = Dx = y = Dy$ when $t = 0$.

72) Solve $(D - 2)x - (D + 1)y = 6e^{3t}; (2D - 3)x + (D - 3)y = 6e^{3t}\ if\ x = 3; y = 0$ when $t = 0$.

73) Solve $(D^2 + 2)x - Dy = 1; Dx + (D^2 + 2)y = 0\ if\ x = Dx = y = Dy = 0$ when $t = 0$.

74) Solve $\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$ where $y\left(\frac{\pi}{2}, t\right) = 0;\ (\frac{\partial y}{\partial x})_x = 0$ and $y(x, 0) = cos5x$.

75) Solve $\frac{\partial y}{\partial t} = 3\frac{\partial^2 y}{\partial x^2}$ where $y_x(0, t) = 0, y\left(\frac{\pi}{2}, t\right) = 0$; and $y(x, 0) = 20cos3x - 5cos9x$.

76) Solve $\frac{\partial y}{\partial t} = 2\frac{\partial^2 y}{\partial x^2}$ where $y(0, t) = 0 = y(5, t)$ and $y(x, 0) = 10sin4\pi x$.

77) Solve $\frac{\partial^2 y}{\partial x^2} - \frac{\partial^2 y}{\partial t^2} = xt$ where $y = 0 = \frac{\partial y}{\partial t}$ at $t = 0$.

78) Solve: $\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}$ if $u_x(0, t) = 0, u(x, 0) = \begin{cases} x & 0 \le x \le 1 \\ 0 & x > 1 \end{cases}$
and $u(x, t)$ is bounded where $x > 0, t > 0$

79) Solve $\frac{\partial U}{\partial t} = 2\frac{\partial^2 U}{\partial x^2}$ if $U(0, t) = 0; U(x, 0) = e^{-x}$; $x > 0, U(x, t)$ is bounded where

$x > 0, t > 0$.

80) Solve: $\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}$, $x > 0, t > 0$ subject to the conditions

i) $u = 0$ when $x = 0, t > 0$.

ii) when $t = 0$; $u = \begin{cases} 1 & 0 < x < 1 \\ 0 & x \ge 1 \end{cases}$

iii) $u(x, t)$ is bounded.