

# DEPARTMENT OF MATHEMATICS

B.A. / B.Sc.-III (Practical) Examination 2010-2011

Subject : MATHEMATICS (New Syllabus)

Paper : IV(a) Numerical Analysis

## QUESTION BANK

Time : 3 hours

Marks : 50

### UNIT-I

- 1) Define the term percentage error. If  $u = 3v^7 - 6v$  Find the percentage error in  $u$  at  $v = 1$ , if the error in  $v$  is 0.05.
- 2) Define the terms absolute and relative errors. If  $y = \frac{0.31x+2.73}{x+0.35}$ , where the coefficients are rounded off. Find the absolute and relative error in  $y$  when  $x = 0.5 \pm 0.1$
- 3) If  $u = \frac{5xy^2}{z^3}$  then find maximum relative error at  $\Delta x = \Delta y = \Delta z = 0.001$  and  $x = y = z = 1$
- 4) Find the real root of  $x^3 - x - 1 = 0$ , using Bisection method.
- 5) Find the real root of  $x^3 - x^2 - 1 = 0$  up to three decimal places using Bisection method.
- 6) Use iterative method to find a real root of the following equation, correct to four decimal places  $x = \frac{1}{(x+1)^2}$ .
- 7) Use iterative method to find a real root of the following equation, correct to four decimal places  $x = (5 - x)^{\frac{1}{3}}$ .
- 8) Use iterative method to find a real root of the following equation, correct upto four decimal places  $\sin x = 10(x - 1)$ .
- 9) Establish the formula  $x_{i+1} = \frac{1}{2}(x_i + \frac{N}{x_i})$  and hence compute the value of  $\sqrt{2}$  correct to six decimal places.  
Use newton Raphson method to obtain a root and correct to three decimal places of the following equations:
  - 10)  $\sin x = 1 - x$
  - 11)  $x^4 + x^2 - 80 = 0$
  - 12)  $3x = \cos x + 1$ .
- 13) Find  $\sqrt[3]{12}$  by Nweton's method.
- 14) Find a double root of  $x^3 - 3x^2 + 4 = 0$  by Generalised Newton's method.
- 15) Using Ramanujan's method find a real root of the equation  $xe^x = 1$ .

16) Find the root of the equation  $\sin x = 1 - x$  by Ramanujan's method.

17) Find the smallest root of the equation  $f(x) = x^3 - 6x^2 + 11x - 6 = 0$ .

18) Using Ramanujan's method, find the real root of the equation

$$1 - x + \frac{x^2}{(2!)^2} - \frac{x^3}{(3!)^2} + \frac{x^4}{(4!)^2} - \dots = 0.$$

19) Find the root of the equation  $f(x) = x^3 - 2x - 5 = 0$  which lies between 2 & 3 by Muller's method.

20) Use Muller's method to find a root of the equation  $x^3 - x - 1 = 0$ .

#### UNIT-I I

21) Using the difference operator prove the following (i)  $\mu = \sqrt{1 + \frac{\delta^2}{4}}$

(ii)  $1 + \mu^2 \delta^2 = (1 + \frac{\delta^2}{2})^2$

22) Find  $u_6$  if  $u_0 = -3, u_1 = 6, u_2 = 8, u_3 = 12$  and 3<sup>rd</sup> differences are constant.

23) Find a cubic polynomial which takes the values

$x$	0	1	2	3	4	5
$y$	1	2	4	8	15	26

24) If  $y_0 = 2649, y_2 = 2707, y_3 = 2967, y_4 = 2950, y_5 = 2696$  and  $y_6 = 2834$  then find  $y_1$ .

25) Prove the following

a)  $u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \dots + \Delta^{n-1} u_{x-n} + \Delta^n u_{x-n}$ .

b)  $u_x + x_{c_1} \Delta^2 u_{x-1} + x_{c_2} \Delta^4 u_{x-2} + \dots = u_0 + x_{c_1} \Delta u_1 + x_{c_2} \Delta^2 u_2 + \dots$

26) From the following table, find the number of students who secured mark between 60 and 70.

Marks obtained	0-40	40-60	60-80	80-100	100-120
Number of students	250	120	100	70	50

27) Find the cubic polynomial which takes the values :

$$y(1) = 24, \quad y(3) = 120, \quad y(5) = 336, \quad y(7) = 720. \quad \text{Hence obtain } y(8).$$

28) The following data gives the melting point of an alloy of lead and zinc;  $\theta$  is the temperature in degree centigrade;  $x$  is the percent of lead. Find  $\theta$  when  $x = 84$ .

$x$	40	50	60	70	80	90
$\theta$	184	204	226	250	276	304

29) From the following table, find the value of  $e^{1.17}$  by using Gauss forward formula.

$x$	1.00	1.05	1.10	1.15	1.20	1.25	1.30
$e^x$	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

30) The following values of  $x$  and  $y$  are given .Find the value of  $y(0.543)$ .

$x$	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$y(x)$	2.631	3.328	4.097	4.944	5.875	6.896	8.013

31) Use Gauss interpolation formula to find  $y_{41}$  with help of following data

$$y_{30} = 3678.2, \quad y_{35} = 2995.1, \quad y_{40} = 2400.1, \quad y_{45} = 1876.2, \quad y_{50} = 1416.2$$

32) By using central difference formula find the value of  $\log 337.5$  satisfying the following table

$x$	310	320	330	340	350	360
$\log x$	2.4014	2.5052	2.5185	2.5315	2.5441	2.5563

33) Values of  $y = \sqrt{x}$  are listed in the following table, which are rounded off to 5 decimal places. Find  $\sqrt{1.12}$  by using Stirling's formula

$x$	1.00	1.05	1.10	1.15	1.20	1.25	1.30
$y = \sqrt{x}$	1.00000	1.02470	1.04881	1.07238	1.09544	1.11803	1.14017

34) By using Lagrange's formula, express the following rational fraction as sum of

partial fractions  $\frac{x^2+6x+1}{(x^2-1)(x^2-10x+24)}$ .

35) By means of Lagrange's formula prove that approximately

$$y_0 = \frac{1}{2} (y_1 + y_{-1}) - \frac{1}{8} \left[ \frac{1}{2} (y_3 - y_1) - \frac{1}{2} (y_{-1} - y_{-3}) \right]$$

36) Apply Lagrange's formula to find the root of  $f(x) = 0$  when

$$f(30) = -30, f(34) = -13, f(38) = 3, f(42) = 18.$$

37) Use Stirling's formula to find  $u_{32}$  for the following table

$$u_{20} = 14.035, u_{25} = 13.674, u_{30} = 13.257, u_{35} = 12.734, u_{40} = 12.089, \\ u_{45} = 11.309.$$

38) Construct the divided difference table for the given data and evaluate  $f(1)$ .

$x$	-4	-2	-1	0	2	5	10
$f(x)$	469	47	7	1	-5	271	7091

39) Use Newton's divided difference interpolation to obtain a polynomial  $f(x)$  satisfying the following data of values and hence find  $f(5)$ .

$x$	-1	0	3	6	7
$f(x)$	3	-6	39	822	1611

40) Prove that the third order divided difference of the function  $f(x) = \frac{1}{x}$  with arguments  $a, b, c, d$  is  $-\frac{1}{abcd}$ .

### UNIT-III

41) Fit a straight line of the form  $y = a + bx$  to the data

$x$	0	5	10	15	20	25	30
$y$	10	14	19	25	31	36	39

42) Find best values of  $a, b, c$  so that the parabola  $y = a + bx + cx^2$  fits the data

$x$	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$y$	1.1	1.2	1.5	2.6	2.8	3.3	4.1

43) Fit a second degree parabola of the  $y = ax^2 + bx + c$  to the following data.

$x$	0	1	2	3	4
$y$	1	5	10	22	38

44) Determine the constants  $a$  and  $b$  by the method of least squares such that  $y = ae^{bx}$  fits the following data:

$x$	2	4	6	8	10
$y$	4.077	11.084	30.128	81.897	222.62

45) Fit a function of the form  $y = ax^b$  to the following data:

$x$	2	4	7	10	20	40	60	80
$y$	43	25	18	13	8	5	3	2

46) Find the values of  $a_0$  and  $a_1$  so that  $y = a_0 + a_1x$  fits the data given in the table

$x$	0	1	2	3	4
$y$	1.0	2.9	4.8	6.7	8.6

47) Find  $\frac{d}{dx} (J_0)$  at  $x = 0.1$  from the data given in the table:

$x$	0.0	0.1	0.2	0.3	0.4
$J_0(x)$	1.0000	0.9975	0.9900	0.9776	0.9604

48) Find the first and second derivatives of  $f(x)$  at the point  $x = 3.0$  from the following table:

$x$	3.0	3.2	3.4	3.6	3.8	4.0
$f(x)$	-14.000	-10.032	-5.296	0.256	6.672	14.000

49) From the following table of values of  $x$  and  $y$  obtain  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  for  $x = 2.2$

$x$	1.0	1.2	1.4	1.6	1.8	2.0	2.2
$y$	2.7183	3.3210	4.0552	4.9530	6.0496	7.3891	9.0250

50) The following table of values of  $x$  and  $y$  is given :

$x$	0	1	2	3	4	5	6
$y$	6.9897	7.4036	7.7815	8.1291	8.4510	8.7506	9.0309

Find  $\frac{dy}{dx}$  at  $x = 3$ .

51) From the following values of  $x$  and  $y$ , find  $\frac{dy}{dx}$  when  $x = 6$ .

$x$	4.5	5.0	5.5	6.0	6.5	7.0	7.5
$y$	9.69	12.90	16.71	21.18	26.37	32.34	39.15

52) Find the minimum and maximum values of the functions from the following table

$x$	0	1	2	3	4	5
$f(x)$	0	0.25	0	2.25	16.00	56.25

53) Evaluate by using Trapezoidal rule

a)  $\int_0^{\pi} t \sin t \, dt$  ( with 6 strips)

b)  $\int_{-2}^2 \frac{t}{5+2t} \, dt$  ( with 8 strips)

54) When a train is moving at 30 miles an hour, steam is burnt off and breaks are applied. The speed of the train in miles per hour after  $t$  seconds is given by :

$t$	0	5	10	15	20	25	30	35	40
$v$	30	24	19.5	16	13.6	11.7	10.8	8.5	7.0

Determine how far the train has moved in 40 seconds.

55) Evaluate  $\int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} \, d\theta$ , using Simpson's rule with  $h = \frac{\pi}{12}$ .

56) Use the Simpson's  $\frac{3^{th}}{8}$  rule to obtain an approximation of  $\int_0^{0.3} (1 - 8x^3)^{\frac{1}{2}} dx$  with  $h=0.05$ .

57) Evaluate  $\int_0^1 \cos x \, dx$  using  $h=0.2$ .

58) Find the value of  $\int_3^7 x^2 \log x \, dx$  by taking 8 strips using Boole's rule.

59) Use Weedle's rule to obtain an approximate value of  $\pi$  from the formula

$$\int_0^1 \frac{1}{1+x^2} \, dx = \frac{\pi}{4}$$

60) Apply Trapezoidal and Simpson's rules to the integral  $I = \int_0^1 \sqrt{1-x^2} \, dx$  by dividing the range into 10 equal parts.

#### UNIT-IV

61) Use Matrix inversion method to solve the system of equation:

$$3x + 2y + 4z = 7, \quad 2x + y + z = 7, \quad x + 3y + 5z = 2.$$

62) Use Matrix inversion method to solve the system of equation:

$$x + 2y + 3z = 10, \quad 2x - 3y + z = 1, \quad 3x + y - 2z = 9.$$

63) Solve the following system of equations using Gauss elimination method:

$$\begin{aligned} x_1 - 2x_2 - x_4 &= 2, & 2x_1 + 2x_2 + x_3 + 2x_4 &= 7 \\ 3x_1 - x_2 - 2x_3 - x_4 &= 3, & x_1 - 2x_4 &= 0. \end{aligned}$$

64) Solve the following system of equations using Gauss elimination method:

$$\begin{aligned} 2x_1 + x_2 + 4x_3 &= 12, & 8x_1 - 3x_2 + 2x_3 &= 20, \\ 4x_1 + 11x_2 - x_3 &= 33 \end{aligned}$$

65) Solve the following system of equations using Factorization method:

$$5x - 2y + z = 4, \quad 7x + y - 5z = 8, \quad 3x + 7y + 4z = 10.$$

66) Solve the following system of equations using Factorization method:

$$2x - 3y + 10z = 3, \quad -x + 4y + 2z = 20, \quad 5x + 2y + z = -12.$$

67) Solve the following system of equations using Jacobi's iterative method:

$$10x + 2y + z = 9, \quad 2x + 20y - 2z = -44, \quad -2x + 3y + 10z = 22.$$

68) Apply Gauss-siedal iterative method to solve:

$$10x + y + z = 12, \quad 2x + 10y + z = 13, \quad 2x + 2y + 10z = 14.$$

69) Apply Gauss-siedal iterative method to solve:

$$27x + 6y - z = 85, \quad 6x + 15y + 2z = 72, \quad x + y + 54z = 110.$$

70) Solve the following system of equations using Jacobi's iterative method:

$$\begin{aligned} 17x_1 + 65x_2 - 13x_3 + 50x_4 &= 84, & 12x_1 + 16x_2 + 37x_3 + 18x_4 &= 25 \\ 56x_1 + 23x_2 + 11x_3 - 19x_4 &= 36, & 3x_1 - 5x_2 + 47x_3 + 10x_4 &= 18. \end{aligned}$$

71) Using Taylor's series method to find the value of  $y(0.1)$  and  $y(0.2)$  if  $y(x)$

satisfies  $\frac{dy}{dx} = x - y^2$  with  $y(0) = 1$ .

72) Solve  $\frac{dy}{dx} = x + y$  by Taylor's series method starting with  $x_0 = 1, y_0 = 0$  and carry to  $x = 1.2$  with  $h = 0.1$ . Compare the final result with the value of explicit solution.

- 73) Using Picard's method solve  $\frac{dy}{dx} = 1 + xy$  with  $y(0) = 1$ . Find  $y(0.1), y(0.2) \dots y(0.5)$ .
- 74) Use Picard's method to approximate  $y$  upto 3 decimal places when  $x = 0.2$ . Given that  $y(0) = 1$  and  $\frac{dy}{dx} = x - y$ .
- 75) Using Euler's method, solve the following initial value problems:
- i)  $\frac{dy}{dx} + 2y = 0, \quad y(0) = 1$
- ii)  $\frac{dy}{dx} - 1 = y^2, \quad y(0) = 0$  in each case take  $h = 0.1$  and obtain  $y(0.1), y(0.2), y(0.3)$ .
- 76) Given  $\frac{dy}{dx} = x^2 + y, \quad y(0) = 1$  determine  $y(0.02), y(0.04), y(0.06)$  using modified Euler's method
- 77) Find  $y$  when  $x = 0.1, x = 0.2, x = 0.3$  from the following initial value problem by Runge-Kutta's 4<sup>th</sup> order method  $y' = x - y^2, \quad y(0) = 1$
- 78) Given  $\frac{dy}{dx} = 1 + y^2$ , where  $y = 0$  when  $x = 0$ , find  $y(0.2), y(0.4), y(0.6)$  by Runge-Kutta's 4<sup>th</sup> order method.
- 79) Apply Milne's method to the equation  $y' = x + y^2$  with  $y(0) = 0$  to find  $y(0.8)$ . (take  $h = 0.2$  to obtain initial values)
- 80) Using Milen's method solve the differential equation  $(1 + x) \frac{dy}{dx} + y = 0$  with  $y(0) = 2$ . Find  $y(1.5)$ . (take  $h = 0.5$  to obtain initial values)