DEPARTMENT OF MATHEMATICS

B.A. / B.Sc.III (Practical) Examination 2010-2011

Subject: MATHEMATICS (New Syllabus)

Paper : III

QUESTION BANK

Time : 3 hours **Marks : 50**

UNIT-I (LINEAR ALGEBRA-I)

1) Let V be the set of all pairs (x,y) of real numbers and let F be the field of real numbers.

Define: $(x,y)+(x_1,y_1)=(x+x_1,0)$& $c(x,y)=(cx,0)$. Is V with these operations a vector space over the field of real numbers?

2) Is the set of all polynomials in x of degree ≤ 2 a vector space ? Justify.

3) Let R be the field of real numbers. Which of the following are subspaces of V_3(R)

(i)$\{(x,2y,3z): x,y,z \in R\}$

(ii) $\{(x,x,x): x \in R\}$

(iii))$\{(x,y,z): x,y,z \textit{ are rational numbers}\}$

4) Which of the following sets of vectors $\alpha=(a_1,a_2,a_3,\dots\dots.a_n)$ in R^n are subspaces of R^n(n $\geq$ 3)?

(i) all α such that $a_1 \leq 0$

(ii) all α such that a_3 is an integer

(iii) all α such that $a_1+a_2+a_3+\dots.+a_n=k$ (k is a given constant).

5) Let V=R^3 and W be the set of all ordered triads (x,y,z) such that $x-3y+4z=0$

Prove that W is a subspace of R^3.

6) In the vector space R^4 determine whether or not the vector (3,9,-4,2) is a linear combination of the vectors (1,-2,0,3),(2,3,0,-1) and (2,-1,2,1).

7) Determine whether the vector (3,-1,0,-1) in the subspace of R^4 spanned by the vectors

(2,-1,3,2), (-1,1,1,-3) and (1,1,9,-5).

8) Find whether the following sets are linearly dependent or independent:

a) {(1,1,-1), (2,-3,5), (-2,1,4)} of R^3

b) $\left\{\begin{bmatrix} 1 & -2 \\ -1 & 4 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 2 & -4 \end{bmatrix}\right\}$ of M_{2x2}(R)

9) Determine whether the following vectors form basis of the given vector spaces

a) (2,1,0), (1,1,0), (4,2,0) of R^3

b) $x^2 + 3x - 2$, $2x^2 + 5x - 3$, $-x^2 - 4x + 4$ of $P_2(R)$

10) a) Show that the vectors (2,1,4), (1,-1,2), (3,1,-2) form the basis of R^3

b) Determine whether or not the vectors : (1,1,2), (1,2,5), (5,3,4) form a basis of R^3.

11) Let V=R^3 and W be the subspace of R^3 given by $w = \{(x, y, z): x - 3y + 4z = 0\}$. Prove that Wis a subspace of R^3 and find its dimension.

12) Let F be the field of complex numbers and let T be function from F^3 $into$ F^3 defined by

T$(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3,\ 2x_1 + x_2 - x_3,\ -x_1 - 2x_2)$. Verify that T is linear transformation and describe the null space of T.

13) Show that the mapping $T: R^2 \longrightarrow R^3$ defined as T(a,b)=(a-b, b-a, -a) is a linear transformation from R^2 $into$ R^3 .Find the Range ,Rank,Nullspace and Nullity of T.

14) Let F be a subfield of complex numbers and let T be the function from $F^3 into\ F^3$ defined by T(a,b,c)= (a-b+2c, 2a+b, -a-2b+2c) show that T is a linear transformation find also the Rank and Nullity of T.

15) Let $T: R^3 \longrightarrow R^3$ be the linear transformation defined by T(x,y,z)=(x+2y-z, y+z, x+y-2z)

Find a basis and dimension of (i) the Range of T (ii) the Nullspace of T

16) Describe explicitly the linear transformation $T: R^2 \longrightarrow R^2$ such that T(2,3)=(4,5) and T(1,0)=(0,0).

17) Let T_1 and T_2 be two linear operators defined on V_3(R) by T_1 (a,b,c)=(a+b, 2b, 2b-a)

T_2 (a,b,c)=(3a, a-b, 2a+b+c) for all (a,b,c) ∈ V_3(R) show that $T_1 T_2 \neq T_2 T_1$

18) Show that the operator T on R^3 defined by T(x,y,z)=(x+z, x-z, y) is invertible and find similar rule defining T^{-1}.

19) Let T be the linear operator on R^3 defined by

T(x_1, x_2, x_3) = $(3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$ What is the matrix of T in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3\}$ where α_1=(1,0,1) α_2=(-1,2,1) and α_3=(2,1,1)?

20) Find the matrices of the linear transformation T on $V_3(R)$ defined as T(a,b,c)=(2b+c, a-4b, 3a) With respect to the standard ordered basis **B**={(1,0,0),(0,1,0),(0,0,1)}, and ordered basis $\boldsymbol{B'}$.={(1,1,1),(1,1,0),(1,0,0)}

UNIT-II(LINEAR ALGEBRA-II)

21) Find all (complex) proper values and proper vectors of the following matrices

a)$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ b)$\begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix}$

22) Let T be the linear operator on R^3 which is represented in the standard ordered basis by the matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Prove that T is diagonalizable.

23) a) Determine whether the matrix A= $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ similar over the field R to a diagonal matrix?

Is A similar over the field C to a diagonal matrix?

b) Prove that the matrix A= $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ is not diagonalizable over the field C.

24) Show that the characteristic equation of the complex matrix $A = \begin{bmatrix} 0 & 0 & c \\ 1 & 0 & b \\ 0 & 1 & a \end{bmatrix}$ is

$x^3 - ax^2 - bx - c = 0.$

25) Find all the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$

26) Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & -2 \\ 3 & -6 & -4 \end{bmatrix}$

27) Show that the distinct eigenvectors of a matrix A corresponding to distinct eigen values of A are linearly independent.

28) If α, β are vectors in an inner product space $V(F)$ and a,b ∈F , then prove that:

i)$\|a\alpha + b\beta\|^2 = |a|^2\|\alpha\|^2 + a\bar{b}(\alpha, \beta) + \bar{a}b(\beta, \alpha) + |b|^2\|\beta\|^2$

ii)$Re(\alpha, \beta) = \frac{1}{4}\|\alpha + \beta\|^2 - \frac{1}{4}\|\alpha - \beta\|^2$.

29) Prove that if α, β are vectors in an unitary space then

(i) $4(\alpha, \beta) = \|\alpha + \beta\|^2 - \|\alpha - \beta\|^2 + i\|\alpha + i\beta\|^2 - i\|\alpha - i\beta\|^2$.

(ii) $(\alpha, \beta) = Re(\alpha, \beta) + iRe(\alpha, i\beta)$

30) If in an inner product space $\|\alpha + \beta\| = \|\alpha\| + \|\beta\|$, then prove that the vectors α, β are linearly dependent. Give an example to show that the converse of this statement is false.

31) If $\alpha = (a_1, a_2\ a_3, \ldots\ldots, a_n)$, $\beta = (b_1\ b_2\ b_3, \ldots\ldots\ldots b_n) \in V_n(R)$ then prove that :

$(\alpha, \beta) = a_1b_1 + a_2b_2 + a_3b_3 + \ldots\ldots + a_nb_n$. defines an inner product on $V_n(R)$.

32) If $\alpha = (a_1, a_2), \beta = (b_1, b_2) \in V_2(R)$. Define : $(\alpha, \beta) = a_1b_1 - a_2b_1 - a_1b_2 + 4a_2b_2$

Show that all the postulates of an inner product hold good.

33) Let V(C)be the vector space of all continuous complex-valued functions on the unit interval, $0 \le t \le 1$. If f(t),g(t) ∈ V,let us define :$\big(f(t), g(t)\big) = \int_0^1 f(t)\overline{g(t)}\,dt$. Show that all the postulates of an inner product hold good.

34) Determine whether the following define an inner product in $V_2(R)$:

$(\alpha, \beta) = 2x_1y_1 + 5x_2y_2$ given by $\alpha = (x_1, x_2), \beta = (y_1, y_2)$

35) Apply Gram-Schmidt process to the vectors $\beta_1 = (1,0,1), \beta_2 = (1,0,-1)\ \beta_3 = (0,3,4)$ To obtain an orthonormal basis for $V_3(R)$ with the standard inner product.

36) Prove that the vectors $\alpha and\ \beta$ in a real inner product space are orthogonal if and only if $\|\alpha + \beta\|^2 = \|\alpha\|^2 + \|\beta\|^2$.

37) Prove that two vectors $\alpha and\ \beta$ in a complex inner product space are orthogonal if and only If $\|a\alpha + b\beta\|^2 = \|a\alpha\|^2 + \|b\beta\|^2$ for all pairs of scalars a and b.

38) a) Find a vector of unit length which is orthogonal to the vector α =(2,-1,6) of $V_3(R)$ with respect to the standard inner product.

b) Find two mutually orthogonal vectors each of which is orthogonal to the vector: α =(4,2,3) of $V_3(R)$ with respect to the standard inner product.

39) Let V be a finite-dimensional inner product space and let $\{ \alpha_1, \alpha_2, \dots \dots \dots \alpha_n\}$ be an orthonormal basis for V. Show that for any vectors α, β in V, $(\alpha, \beta) = \sum_{k=1}^{n} (\alpha, \alpha_k)\overline{(\beta, \alpha_k)}$.

40) Given the basis (2,0,1),(3,-1,5) and (0,4,2) for $V_3(R)$, construct from it by the Gram-Schmidt process an orthonormal basis relative to the standard inner product.

UNIT-III (MULTIPLE INTEGRALS)

41)Evaluate the following integral: $\iint xy(x^2+y^2)dxdy$ over $[(0,a;0,b)]$

42) Evaluate the following integral: $\iint \frac{x-y}{x+y}dxdy$ over $[0,1;0,1]$.

43) Show that $\int_0^1\{\int_0^1 \frac{x^2-y^2}{x^2+y^2}dy\}dx = \int_0^1\{\int_0^1 \frac{x^2-y^2}{x^2+y^2}dx\}dy$.

44) Evaluate $\iint x^2y^2\,dxdy$ over the domain $\{(x,y): x \geq 0, y \geq 0, (x^2+y^2) \leq 1\}$.

45) Evaluate the following integral: $\iint (x^2+y^2)dxdy$ over the domain bounded by $xy = 1, y = 0, y = x, x = 2$.

46) Evaluate the following integral: $\iint \frac{x^2}{y^2}dxdy$ over the domain bounded by $xy = 1, y = x, x = 2$.

47) Show that $\int_0^1\{\int_0^1 f(x,y)dx\}dy \neq \int_0^1\{\int_0^1 f(x,y)dy\}dx$. Where $f(x,y) = \frac{y^2-x^2}{(y^2+x^2)^2}$.

48)) Evaluate $\int_2^4 \int_{\frac{4}{x}}^{\frac{20-4x}{8-x}} (4-y)dydx$ and also change the order of integration.

49) Evaluate $\iint_R (x^2+y^2)dxdy$ over the domain bounded by $y = x^2$ and $y^2 = x$.

50) Prove that $\int_0^1 dx \int_x^{\frac{1}{x}} \frac{ydy}{(1+xy)^2(1+y^2)} = \frac{\pi-1}{4}$.

51) Evaluate $\iint (x+y+a)\,dxdy$ taken over $\{(x,y): (x^2+y^2) \leq a^2\}$

52) Evaluate $\iint_R \sqrt{4x^2-y^2}dxdy$ where the domain R is the triangle bounded by the lines

$y = 0, y = x, x = 1.$

53) Verify that $\iint_R (x^2+y^2)dydx = \iint_R (x^2+y^2)dxdy$ where the domain R is the triangle bounded by the lines $y = 0, y = x, x = 1.$

54) Evaluate $\iint_R f(x,y)dxdy$ where $f(x,y) = x^2 + y^2$ and $R = \{(x,y): y = x^2, x = 2, y = 1\}.$

55) Evaluate $\iint_R xy(x+y)dxdy$ R is the region between $y = x^2, y = x.$

56) Prove that $\int_0^1 dx \int_0^1 \frac{(x-y)dy}{(x+y)^3} = \frac{1}{2}$ and $\int_0^1 dy \int_0^1 \frac{(x-y)dx}{(x+y)^3} = \frac{-1}{2}$.

57) Evaluate $\iint_R x\left(e^{x^2-y^2}\right)dxdy$ where R is closed region bounded by the lines $y = x, y = x - 1,$ $y = 0, y = 1.$

58) Evaluate $\iint_R ydxdy$ over part of the plane bounded by the lines $y = x$ and the parabola $y = 4x - x^2.$

59)Change the order of integration and evaluate $\int_0^a \int_{\frac{x}{a}}^{\sqrt{\frac{x}{a}}} (x^2+y^2)dydx.$

60)Evaluate by changing order of integration $\int_0^a \int_0^{\sqrt{x^2-y^2}} (xy)dxdy.$

UNIT-IV (VECTOR CALCULUS)

61) a) If $A = 5t^2 i + tj - t^3 k$ and $B = sint\ i - cost\ j$ then find $\frac{d}{dt}(A \times B)$

b)If $A = x^2yzi - 2xz^3j + xz^2k$ and $B = 2zi + yj - x^2k$ Find $\frac{\partial^2}{\partial x \partial y}(A \times B)$ at (1,0,-2)

62) If $\vec{r} = xi + yj + zk$ and r = $|\vec{r}|$ then find : a) ∇r^n b) div(grad r)

63) If $A = 2yzi - x^2yj + xz^2k$ and $\varphi = 2x^2yz^3$ find: (i)$(A \times \nabla)\ \varphi$ (ii) $A \times \nabla\varphi$.

Are they equal?

64) a) For $\varphi = 2x^3y^2z^4$ find div(grad φ)

b) If $A = x^2yi - 2xzj + 2yzk$,Find curl(curl A)

65) a)Define Solenoidal vector . Show that $A = (2x^2 + 8xy^2z)i + (3x^3y - 3xy)j - (4y^2z^2 + 2x^3z)k$ is not a Solenoidal, but $B = xyz^2A$ is Solenoidal.

b) Define irrotational vectors. If V =(x+2y+az)i+(bx-3y-z)j+(4x+cy+2z)k is irrotational then find a,b,c.

66) a) If $\vec{r} = xi + yj + zk$ and r =$|\vec{r}|$ then prove that $\frac{\vec{r}}{r^2}$ is irrotational

b) If A and B are irrotational , then prove that $(A \times B)$ Solenoidal

67) If $U = 3x^2y, V = xz^2 - 2y$ evaluate a) grad [grad U.grad V] b) curl [grad U × grad V].

68) Prove that : a)$\nabla^2 f(r) = \frac{d^2f}{dr^2} + \frac{2}{r}\frac{df}{dr}$.

b) Find $f(r)$ such that $\nabla^2 f(r) = 0$ where r =$|\vec{r}|$ & $\vec{r} = xi + yj + zk$.

69) The acceleration of a particle at any time t≥ 0 is given by :

a=$\frac{dv}{dt} = 12\cos 2ti - \sin 2tj + 16tk$. If the velocity V and displacement r are zero at t=0, find v and r at any time.

70) a) Find the total work done in moving a particle in a force field given by F= 3xyi-5zj+10xk along the curve x=$t^2 + 1$, y= $2t^2$, z= t^3 from t=1 to t=2.

b) Find the work done in moving a particle one moving around a circle C in the XY-plane , if the circle has center at origin and radius 3 and if the force field is given by :

$F = (2x - y + z)i + (x + y - z^2)j + (3x - 2y + 4z)k.$

71) Find $\int_c \vec{F}.d\vec{r}$ For $\vec{F} = (2xy + z^3)i + x^2j + 3xz^2k$ where C is the line joining (1,-2,1) to (3,1,4).

72) Evaluate $\iint_S A.n\, ds,$ where A=18zi-12j+3yk and S is that part of the plane 2x+3y+6z=12 which is located in first octant.

73) If $\vec{F}$=$(2x^2 - 3z)i - 2xyj - 4xk$ evaluate : a) $\iiint_V \nabla.$Fdv and b) $\iiint_V \nabla \times \vec{F}dv$,where V is a closed bounded by the plane x=0=y=z and 2x+2y+z=4.

74) Evaluate $\iint A.n\, ds\, for\, A = yi + 2xj - zk$ and S is the surface of the plane 2x+y=6 in the first octant cut off by the plane z=4.

75) Verify Green's theorem in a plane for $\oint_C (xy + y^2)dx + x^2dy$ where c is the closed curve of the region bounded by :y=x and y=x^2.

76) Verify Gauss divergence theorem for $\vec{F} = 4xzi - y^2j$+yzk and S is the surface of the cube bounded by x=0,x=1;y=0,y=1;z=0,z=1.

77) Verify Green's theorem in a plane for $\oint_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the boundary of the region defined by $y = \sqrt{x}$, y=x^2.

78) Verify Green's theorem in a plane for $\oint_c (x^2 - 2xy)dx + (x^2y + 3)dy$ around the boundary of the region defined by $y^2 = 8x \text{ and } x = 2$.

79) Verify Stoke's theorem $A = (2x - y)i - yz^2j - y^2zk$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is it's boundary.

80) Verify divergence theorem for $A = 2x^2yi - y^2j + 4xz^2k$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and $x = 2$.