90 hrs
(3 hrs/ week)

Mathematics: Paper – III
LINEAR ALGEBRA AND VECTOR CALCULUS

Part A: Linear Algebra

Unit-I: (25 Hours)

Vector spaces, General properties of vector spaces, Vector subspaces, Algebra of subspaces, linear combination of vectors. Linear span, linear sum of two subspaces, Linear independence and dependence of vectors, Basis of vector space, Finite dimensional vector spaces, Dimension of a vector space, Dimension of a subspace. Linear transformations, linear operators, Range and null space of linear transformation, Rank and nullity of linear transformations, Linear transformations as vectors, Product of linear transformations, Invertible linear transformation.

Unit-II: (20 Hours)

The adjoint or transpose of a linear transformation, Sylvester's law of nullity, characteristic values and characteristic vectors, Cayley- Hamilton theorem, Diagonalizable operators. Inner product spaces, Euclidean and unitary spaces, Norm or length of a vector, Schwartz inequality, Orthogonality, Orthonormal set, complete orthonormal set, Gram - Schmidt orthogonalisation process.

Prescribed text book:
Linear Algebra by J.N.Sharma and A.R.Vasista, Krishna Prakasham Mandir, Meerut-250002.

Reference Books:
1. Linear Algebra by Kenneth Hoffman and Ray Kunze, Pearson Education (low priced edition), New Delhi
2. Linear Algebra by Stephen H. Friedberg et al Prentice Hall of India Pvt. Ltd. 4^{th} edition 2007

Part B : Multiple integrals and Vector Calculus

Unit-III: (25 Hours)

Multiple integrals: Introduction, the concept of a plane, Curve, line integral- Sufficient condition for the existence of the integral. The area of a subset of R^2, Calculation of double integrals, Jordan curve, Area, Change of the order of integration, Double integral as a limit, Change of variable in a double integration. Lengths of Curves, surface areas, Integral expression for the length of a curve, surfaces, surface areas.

Prescribed book: A Course of Mathematical Analysis by Santhi Narayana and P.K.Mittal, S. Chand Publications. Chapters 16 and 17.

Unit-IV: (20 Hours)

Vector differentiation. Ordinary derivatives of vectors, Space curves, Continuity, Differentiability, Gradient, Divergence, Curl operators, Formulae involving these operators. Vector integration, Theorems of Gauss and Stokes, Green's theorem in plane and applications of these theorems.

Prescribed text Book:

Vector Analysis by Murray. R.Spiegel, Schaum Series Publishing Company, Chapters 3, 4,5,6, and 7.

Reference Books:

1.Text book of vector Analysis by Shanti Narayana and P. K. Mittal,

S. Chand & Company Ltd, New Delhi.

2. Mathematical Analysis by S.C. Mallik and Savitha Arora, Wiley Eastern Ltd.

Mathematics: Paper IV(a) **90 hrs**
NUMERICAL ANALYSIS **(3 hrs/ week)**

UNIT-I: (20 hours)

Errors in Numerical Computations: Numbers and their Accuracy, Errors and their Computation, Absolute, Relative and percentage errors, A general error formula, Error in a series approximation.

Solution of Algebraic and Transcendental Equations: The bisection method, The iteration method, The method of false position, Newton-Raphson method, Generalized Newton-Raphson method, Ramanujan's method, Muller's method

UNIT-II: (25 hours)

Interpolation : Errors in polynomial interpolation, Forward differences, Backward differences, Central Differences, Symbolic relations, Detection of errors by use of D.Tables, Differences of a polynomial, Newton's formulae for interpolation formulae, Gauss's central difference formula, Stirling's central difference formula, Interpolation with unevenly spaced points, Lagrange's formula, Error in Lagrange's formula, Derivation of governing equations, End conditions, Divided differences and their properties, Newton's general interpolation.

UNIT-III: (20 hours)

Curve Fitting: Least-Squares curve fitting procedures, fitting a straight line, nonlinear curve fitting, Curve fitting by a sum of exponentials

Numerical Differentiation and Numerical Integration: Numerical differentiation, Errors in numerical differentiation, Maximum and minimum values of a tabulated function, Numerical integration, Trapezoidal rule, Simpson's 1/3 – rule, Simpson's 3/8 – rule, Boole's and Weddle's rule.

UNIT-IV: (25 hours)

Linear systems of equations, Solution of linear systems – Direct methods, Matrix inversion method, Gaussian elimination method, Method of factorization, Ill-conditioned linear systems. Iterative methods: Jacobi's method, Gauss-siedal method,

Numerical solution of ordinary differential equations : Introduction, Solution by Taylor's Series, Picard's method of successive approximations, Euler's method, Modified Euler's method, Runge – Kutta methods, Predictor – Corrector methods, Milne's method.

Prescribed text Book: Scope as in Introductory Methods of Numerical Analysis by S.S. Sastry, Prentice Hall India (4^{th} Edition.), Chapter - 1 (1. 2, 1. 4, 1, 1.5, 1.6); Chapter - 2 (2.2 – 2.7); Chapter - 3 (3.2, 3.3, 3.7.2, 3.9.1, 3.9.2, 3.10.1, 3.10.2); Chapter - 4. (4.2); Chapter.- 5(5.2 – 5.4.5); Chapter - 6 (6.3.2, 6.3.4, 6.3.7, 6.4); Chapter - 7 (7.2-7.5; 7.6.2).

Reference Books:

1. Numerical Analysis by G. Shankar Rao, New Age International Publishers, Hyderabad.
2. Finite Differences and Numerical Analysis by H.C. Saxena S. Chand and Company, New Delhi

Mathematics: Paper IV(b)
FOURIER SERIES AND INTEGRAL TRANSFORMS

90 hrs
(3 hrs/ week)

UNIT - I: (20 hours)
Fourier series :
Fourier series, Theorems, Dirichlet's conditions, Fourier series for even and odd functions, Half range Fourier series, Other forms of Fourier series
Prescribed text Book: Scope as in ***A course of Mathematical Analysis*** by Shanthi Narayan and P.K Mittal, Published by S. Chand & Company, Chapter 10.

UNIT - II: (25 hours**)**

Laplace transforms:
Definition of Laplace transform, linearity property- Piecewise continuous function.
Existence of Laplace transform, Functions of exponential order and of class A. First and second shifting theorems of Laplace transform, Change of scale property- Laplace transform of derivatives, Initial value problems, Laplace transform of integrals, Multiplication by *t*, Division by *t*, Laplace transform of periodic functions and error function, Beta function and Gamma functions. Definition of Inverse Laplace transform, Linearity property, First and second shifting theorems of inverse Laplace transform, Change of scale property, Division by p, Convolution theorem, Heaviside's expansion formula (with proofs and applications).

UNIT - III: (25 hours)

Fourier transforms : Dirichlet's conditions, Fourier integral formula (without proof), Fourier transform, Inverse Theorem for Fourier transform, Fourier sine and cosine transforms and their inversion formulae. Linearity property of Fourier transforms, Change of scale property, Shifting theorem, Modulation theorem, Convolution theorem of Fourier transforms, Parseval's identity, Finite Fourier sine transform, Inversion formula for sine transform, Finite Fourier cosine Transform, Inversion formula for cosine transform.

UNIT - IV: (20hours)

Applications of Laplace and Fourier transforms :
Applications of Laplace transforms to the solution of ordinary differential equations with constant coefficients and variable coefficients, Simultaneous

ordinary differential equations, Partial differential equations. Applications of Fourier transforms to initial and boundary value problems.

Prescribed text Book: Scope as in ***Integral transforms*** by A.R. Vasistha & Dr. R.K. Gupta Published by Krishna Prakashan Media Pvt. Ltd. Meerut.
Chapter I, Chapter II: all sections except 2.3 and 2.18; Chapter III: sections 3.1, 3.2, 3.3, 3.4; Chapter VI: Sections 6.1 to 6.20 except 6.16; Chapter VII: Sections 7.1 to 7.4; Chapter VIII: Section 8.2.)
Reference Book: Operational Mathematics by R.V.Churchil, McGraw Hill Company

Mathematics: Paper – IV(c)
MECHANICS

90 hrs
(3 hrs/ week)

Part A: STATICS

Unit-I: (25 hours)

Introduction, Composition and resolution of forces, Parallelogram law of forces, Triangle law of forces, Lamis theorem, Polygon of forces, $\lambda - \mu$ theorem, Resultant of a finite number of coplanar forces acting upon a particle, Conditions of equilibrium, Parallel forces, resultant of two parallel forces acting upon a rigid body, Moments, Moments of a force about a point and about an axis, Generalized theorem of moments.

Unit-II: (20 hours)

Couples, Equilibrium of a rigid body acted on by three coplanar forces, General conditions of equilibrium of a rigid body under coplanar forces, Friction, Laws of friction, Limiting friction, Coefficient of friction and simple problems.

Part B: DYNAMICS

Unit-III: (25 hours)

Velocity, Relative velocity, Acceleration, Parallelogram laws of acceleration, Motion under gravity, Newton's laws of motion and their applications to simple problems. Impulse, Work, Energy. Kinetic and potential energies of a body, Principle of conservation of energy.

Unit-IV: (20 hours)

Projectiles, Range on an inclined plane, Collision of elastic bodies, Newton's experimental law, Impact of sphere on a plane, Direct and oblique impact of two spheres, Loss of kinetic energy by impact, Simple harmonic motion, Examples of simple harmonic motion, Simple pendulum, Simple equivalent pendulum.

Prescribed text books:

1. The elements of Statics and Dynamics, Part-I – Statics by S.L. Loney, Book palace, New Delhi.
2. The elements of Statics and Dynamics, Part-II-Dynamics by S.L.Loney, AITBS

Publications and distributions (Regd), Delhi

Reference Book:

Mechanics by P. Durai Pandian, Laxmi Durai Pandian, Muthamizh Jaya Prakasan, S. Chand and Company limited.

Mathematics - Paper –IV(d) **90 hrs**
(3 hrs/ week)

DISCRETE MATHEMATICS

UNIT – I: (15 hours)

Sets and Operations of sets, Relations and functions, Some methods of proof and problem solving strategies, Fundamentals of logic, Logical inferences, Methods of proof of an implication, First order logic and other methods of proof, Rules of inference for quantified prepositions, Mathematical induction.

UNIT – II: **(**25 hours)

Recurrence Relations:

Generating functions of sequences, Calculating coefficients of generating functions; Recurrence relations; Solving recurrence relations by substitution and generating functions; The method of characteristic roots; Solutions of in-homogenous linear recurrence relations.

UNIT – III: **(**25 hours)

Graph Theory:

Basic concepts, Isomorphisms and subgraphs, Trees and their properties, Spanning trees, Directed trees, Binary trees, Planar graphs, Euler's formula, Multi graphs and Euler circuits, Hamiltonian graphs, Chromatic numbers. Four – color problem.

UNIT – IV: **(**25 hours)

Boolean Algebra:
Relations, Properties of special binary relations, Equivalence relations, Ordering relations, Lattices and enumerations , Boolean algebra, Boolean functions, Switching mechanism, Minimization of Boolean functions.

Prescribed text book Scope as in Discrete Mathematics for computer Scientists and Mathematicians by Joe L.Mott, Abraham Kandel, Theodore P.Baker, Printice-Hall of India (Second edition) Chapters 1, 3, 4, 5, 6.

Reference text Books:

1. Discrete Mathematical structures by Bernard Kolman, Robert C. Busby, Sharon Ross, Prentice- Hall of India

2. Discrete Mathematics and its applications by Kenneth H.Rosen, Tata McGraw- Hill.

Mathematics: Paper – IV(e) **90 hrs**
NUMBER THEORY **(3 hrs/ week)**

UNIT – I : (25 hours)

The division algorithm, The greatest common divisor, The Euclid algorithm, The Diophantine equation ax + by = c, Basic properties of congruence, Special divisibility tests, Linear congruencies

UNIT – II : (20 hours)

The Little Fermat's theorem , Wilson's theorem, The functions τ and σ, The Möbius inversion formula, The greatest integer function

UNIT – III : (25 hours)

Euler's Phi-function, Ruler's theorem, Some properties of the Phi-function, Ruler's criterion, The Legendre's symbol and its properties, Quadratic reciprocity

UNIT – IV : (20 hours)

The search for perfect numbers, Mersenne primes, Fermat numbers, Sums of two squares, Sums of more than two squares

Prescribed text Book: Scope as in Elementary Number Theory – David M. Burton,

2^{nd} Edition, Universal book Stall, New Delhi.

Reference Books:

1. An Introduction to the Theory of Numbers – I. Niven and H. Zuckerman 1980, 4^{th} Edition, John Wiley & Sons, New York.

2. Elementary Number Theory & Its Applications – Kenneth Rosen 1987, 2^{nd} Edition, Reading Mass Addison – Wesley.

Model Question Paper (Theory)

B.A/B.Sc. III Year Examination, March/April 2011

MATHEMATICS PAPER-III

Time:3Hrs **Maximum Marks:100**

NOTE: Answer 6 questions from Section- A and 4 questions from Section –B choosing atleast one from each unit. Each question in Section- A carries 6 marks and each question in Section-B carries 16 marks.

SECTION-A (6×6=36)

UNIT-I

1) Define a subspace. Prove that the intersection of two subspaces is again a subspace.

2) Define Linear transformation. Show that the mapping $T: V_3(R) \to V_2(R)$ defined as $T(a_1, a_2, a_3) = (3a_1 - 2a_2 + a_3,\ a_1 - 3a_2 - 2a_3)$ is a linear transformation from $V_3(R)$ in to $V_2(R)$.

UNIT-II

3) Find all eigen values of the matrix $\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$.

4) Define orthogonal set. Show that any orthogonal set of non-zero vectors in an inner product space V is linearly independent.

UNIT-III

5) Evaluate $\iint x^2y^2\,dxdy$ over the domain $\{(x, y): x \geq 0; y \geq 0; (x^2 + y^2) \leq 1\}$.

6) Evaluate $\iint (x^2 + y^2)\,dxdy$ over the domain bounded by $xy = 1; y = 0; y = x; x = 2$.

UNIT-IV

7) Define irrotational vector. Show that $A = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is Irrotational. Find φ such that $A = \nabla\varphi$.

8) Evaluate $\iint A.n\ ds$ where A=18zi-12j+3yk and S is that part of the plane 2x+3y+6z=12 which is located in first octant.

SECTION-B (4×16=64)

UNIT-I

9) a) Define Basis of a vector space. Prove that any two basis of a finite dimensional vector Space $V(F)$ have same number of elements.

b) If W_1,W_2 are two subspaces of a finite dimensional vector Space $V(F)$ then $dim(W_1 + W_2) = dimW_1 + dimW_2 - \dim(W_1 \cap W_2)$.

10) a) State and prove Rank and Nullity theorem in linear transformation.

b) Show that linear operator T defined on R^3 by $T(x, y, z) = (x + z, x - z, y)$ is invertible. And hence find T^{-1}.

UNIT-II

11) a) Prove that distinct characteristic vectors of T corresponding to distinct characteristic of T are linearly independent.

b) Let T be the linear operator on R^3 which is represented in standard ordered basis by the matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Prove that T is diagonalizable.

12) a) State and prove schwarz's inequality .

b) Apply the Gram- Schmidt process to the vector $\beta_1 = (1,0,1)$; $\beta_2 = (1,0,-1)$; $\beta_3 = (0,3,4)$ to obtain an orthonormal basis for $V_3(R)$ with the standard inner product.

UNIT-III

13) a) Prove the sufficient condition for the existence of the integral.

b) Verify that $\iint_R (x^2 + y^2)dydx = \iint_R (x^2 + y^2)dxdy$ where the domain R is the triangle bounded by the lines $y = 0, y = x, x = 1$.

14)a) Prove the equivalence if a double integra with repeated integrals.

b) Evaluate the following integral: $\iint \frac{x-y}{x+y} dxdy$ over $[0,1; 0,1]$.

UNIT-IV

15) a) For any vector A, Prove that $\nabla \times (\nabla \times A) = \nabla(\nabla . \mathrm{A}) - \nabla^2 A$.

b) If $U = 3x^2y; V = xz^2 - 2y$. Evaluate $grad\ [\ (gradU)\ .\ (gradV)\ ]$.

16) a) State and prove Green's theorem in a plane.

b) Verify stoke's theorem for $A = (2x - y)i - yz^2j - y^2zk$. where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the boundary.