

Model Question Paper (Theory)
B.A/B.Sc. III Year Examination, March/April 2011
MATHEMATICS PAPER IV(b)
INTEGRAL TRANSFORMS

Time:3Hrs

Maximum Marks:100

NOTE: Answer 6 questions from Section –A and 4 questions from Section –B choosing atleast one from each unit. Each question in Section –A carries 6 marks and each question in Section-B carries 16 marks.

SECTION-A (6×6=36)

Unit I

- 1) Find the Fourier series of the function $f(x)$ defined on the interval $-\pi < x < \pi$ as follows $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 \leq x < \pi \end{cases}$.
- 2) Explain half range expansion of Fourier cosine series .

Unit II

- 3) Define Laplace transform and find $L(\sin at)$
- 4) Find $L^{-1}\left\{\frac{1}{p(p^2+a^2)}\right\}$

Unit III

- 5) Find fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$.
- 6) Find finite Fourier cosine transform of $2x$.

Unit IV

- 7) Solve $(D^2 + 2D + 1)y = 3te^{-t}$, $t > 0$ subject to conditions $y=4, Dy=2$ when $t=0$.
- 8) Solve $\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$, $y(x,0)=3\sin 2\pi x$, $y(0,t)=0 = y(1,t)$, $0 < x < 1, t > 0$.

SECTION-B(4×16=64)

Unit I

- 9) a) Find the Fourier Series to represent x^2 from $x = -\pi$ to $x = \pi$ and hence obtain the Series: $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$.
b) Expand $f(x) = \begin{cases} \frac{1}{4} - x, & \text{if } 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \text{if } \frac{1}{2} < x < 1 \end{cases}$
- 10) a) Find Fourier Series for $f(x)$ in the interval $(-\pi, \pi)$ where $f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$

b) Express $f(x) = \frac{1}{2}(\pi - x)$ as a fourier series with period 2π to be valid in the interval 0 to 2π .

Unit II

11) a) State and prove change of scale property.

b) Find $L^{-1}\left\{\frac{1}{(p+2)^2(p-2)}\right\}$ Using convolution theorem.

12) a) Find $L\{\sin \sqrt{t}\}$

b) State Heavisides expansion theorem, using it find $L^{-1}\left\{\frac{19p+37}{(p+1)(p-2)(p+3)}\right\}$

Unit III

13) a) State and prove modulation theorem in fourier transform.

b) Find fourier transform of $\frac{e^{-ax}}{x}$

14) a) Find finite cosine transform of $\left(1 - \frac{x}{\pi}\right)$

b) Show that the finite sine transform of $\frac{x}{\pi}$ is $(-1)^{p+1} \frac{1}{p}$.

Unit IV

15) a) solve: $(D^2 - 1)x + 5Dy = t$

$-2Dx + (D^2 - 4)y = -2$ if $x = 0 = Dx = Dy$ when $t = 0$.

b) Using Laplacetransform solve $\frac{\partial y}{\partial t} = 3 \frac{\partial^2 y}{\partial x^2}$ where

$y\left(\frac{\pi}{2}, t\right) = 0; \left(\frac{\partial y}{\partial x}\right)_{x=0} = 0$ and $y(x, 0) = 30\cos 5x$.

16) a) Solve $\frac{\partial U}{\partial t} = 2 \frac{\partial^2 U}{\partial x^2}$ if $U(0, t) = 0; U(x, 0) = e^{-x}; x > 0, U(x, t)$ is bounded where $x > 0, t > 0$.

b) Solve $(D^2 + 9)y = \cos 2t$ if $y(0) = 1; y\left(\frac{\pi}{2}\right) = -1$.