Model Question Paper (Theory)
B.A/B.Sc. III Year Examination, March/April 2011
MATHEMATICS PAPER-III

Time:3Hrs **Maximum Marks:100**

NOTE: Answer 6 questions from Section- A and 4 questions from Section –B choosing atleast one from each unit. Each question in Section- A carries 6 marks and each question in Section-B carries 16 marks.

SECTION-A (6×6=36)

UNIT-I

1) Define a subspace. Prove that the intersection of two subspaces is again a subspace.

2) Define Linear transformation. Show that the mapping $T: V_3(R) \to V_2(R)$ defined as

$T(a_1, a_2, a_3) = (3a_1 - 2a_2 + a_3,\ a_1 - 3a_2 - 2a_3)$ is a linear transformation from $V_3(R)$ in to $V_2(R)$.

UNIT-II

3) Find all eigen values of the matrix $\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$.

4) Define orthogonal set. Show that any orthogonal set of non-zero vectors in an inner product space V is linearly independent.

UNIT-III

5) Evaluate $\iint x^2 y^2\, dxdy$ over the domain $\{ (x, y): x \geq 0; y \geq 0; (x^2 + y^2) \leq 1\}$.

6) Evaluate $\iint (x^2 + y^2)\, dxdy$ over the domain bounded by $xy = 1; y = 0; y = x; x = 2$.

UNIT-IV

7) Define irrotational vector. Show that $A = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is Irrotational. Find φ such that $A = \nabla\varphi$.

8) Evaluate $\iint A.n\, ds$ where A=18zi-12j+3yk and S is that part of the plane 2x+3y+6z=12 which is located in first octant.

SECTION-B (4×16=64)

UNIT-I

9) a) Define Basis of a vector space. Prove that any two basis of a finite dimensional vector Space $V(F)$ have same number of elements.

b) If W_1,W_2 are two subspaces of a finite dimensional vector Space $V(F)$ then $dim(W_1 + W_2) = dimW_1 + dimW_2 - \dim(W_1 \cap W_2)$.

10) a) State and prove Rank and Nullity theorem in linear transformation.

b) Show that linear operator T defined on R^3 by $T(x, y, z) = (x + z, x - z, y)$ is invertible. And hence find T^{-1}.

UNIT-II

11) a) Prove that distinct characteristic vectors of T corresponding to distinct characteristic of T are linearly independent.

b) Let T be the linear operator on R^3 which is represented in standard ordered basis by the matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Prove that T is diagonalizable.

12) a) State and prove schwarz's inequality .

b) Apply the Gram- Schmidt process to the vector $\beta_1 = (1,0,1)$; $\beta_2 = (1,0,-1)$; $\beta_3 = (0,3,4)$ to obtain an orthonormal basis for $V_3(R)$ with the standard inner product.

UNIT-III

13) a) Prove the sufficient condition for the existence of the integral.

b) Verify that $\iint_R (x^2 + y^2)dydx = \iint_R (x^2 + y^2)dxdy$ where the domain R is the triangle bounded by the lines $y = 0, y = x, x = 1$.

14)a) Prove the equivalence if a double integra with repeated integrals.

b) Evaluate the following integral: $\iint \frac{x-y}{x+y} dxdy$ over $[0,1; 0,1]$.

UNIT-IV

15) a) For any vector A, Prove that $\nabla \times (\nabla \times A) = \nabla(\nabla . \mathrm{A}) - \nabla^2 A$.

b) If $U = 3x^2y; V = xz^2 - 2y$. Evaluate $grad\ [\ (gradU) . (gradV)\ ]$.

16) a) State and prove Green's theorem in a plane.

b) Verify stoke's theorem for $A = (2x - y)i - yz^2j - y^2zk$. where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the boundary.

DEPARTMENT OF MATHEMATICS

B.A. / B.Sc.-III (Practical) Examination 2010-2011

Subject : MATHEMATICS (New Syllabus)

Paper : IV(a) Numerical Analysis

QUESTION BANK

Time : 3 hours **Marks : 50**

UNIT-I

1) Define the term percentage error. If $u = 3v^7 - 6v$ Find the percentage error in u at $v = 1$, if the error in v is 0.05.
2) Define the terms absolute and relative errors. If $y = \frac{0.31x+2.73}{x+0.35}$, where the coefficients are rounded off. Find the absolute and relative error in y when $x = 0.5 \pm 0.1$
3) If $u = \frac{5xy^2}{z^3}$ then find maximum relative error at $\Delta x = \Delta y = \Delta z = 0.001$ and $x = y = z = 1$
4) Find the real root of $x^3 - x - 1 = 0$, using Bisection method.
5) Find the real root of $x^3 - x^2 - 1 = 0$ up to three decimal places using Bisection method.
6) Use iterative method to find a real root of the following equation, correct to four decimal places $x = \frac{1}{(x+1)^2}$.
7) Use iterative method to find a real root of the following equation, correct to four decimal places $x = (5-x)^{\frac{1}{3}}$.
8) Use iterative method to find a real root of the following equation, correct upto four decimal places $sinx = 10(x-1)$.
9) Establish the formula $x_{i+1} = \frac{1}{2}(x_i + \frac{N}{x_i})$ and hence compute the value of $\sqrt{2}$ correct to six decimal places.

Use newton Raphson method to obtain a root and correct to three decimal places of the following equations:

10) $sinx = 1 - x$ 11) $x^4 + x^2 - 80 = 0$ 12) $3x = cosx + 1$.

13) Find $\sqrt[3]{12}$ by Nweton's method.

14) Find a double root of $x^3 - 3x^2 + 4 = 0$ by Generalised Newton's method.

15) Using Ramanujan's method find a real root of the equation $xe^x = 1$.

16) Find the root of the equation $sinx = 1 - x$ by Ramanujan's method.

17) Find the smallest root of the equation $f(x) = x^3 - 6x^2 + 11x - 6 = 0$.

18) Using Ramanujan's method, find the real root of the equation

$$1 - x + \frac{x^2}{(2!)^2} - \frac{x^3}{(3!)^2} + \frac{x^4}{(4!)^2} - \cdots = 0.$$

19) Find the root of the equation $f(x) = x^3 - 2x - 5 = 0$ which lies between 2 & 3 by Muller's method.

20) Use Muller's method to find a root of the equation $x^3 - x - 1 = 0$.

UNIT-I I

21) Using the difference operator prove the following (i) $\mu = \sqrt{1 + \frac{\delta^2}{4}}$

(ii) $1 + \mu^2\delta^2 = (1 + \frac{\delta^2}{2})^2$

22) Find u_6 if $u_0 = -3, u_1 = 6, u_2 = 8, u_3 = 12$ and 3rd differences are constant.

23) Find a cubic polynomial which takes the values

x	0	1	2	3	4	5
y	1	2	4	8	15	26

24) If $y_0 = 2649, \ y_2 = 2707, \ y_3 = 2967, \ y_4 = 2950, \ y_5 = 2696$ and $y_6 = 2834$ then find y_1.

25) Prove the following

a) $u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \cdots + \Delta^{n-1} u_{x-n} + \Delta^n u_{x-n.}$

b) $u_x + x_{c_1}\Delta^2 u_{x-1} + x_{c_2}\Delta^4 u_{x-2} + \cdots = u_0 + x_{c_1}\Delta u_1 + x_{c_2}\Delta^2 u_2 + \cdots$

26) From the following table ,find the number of students who secured mark between 60 and 70.

Marks obtained	0-40	40-60	60-80	80-100	100-120
Number of students	250	120	100	70	50

27) Find the cubic polynomial which takes the values :

$y(1) = 24,\ \ y(3) = 120,\ \ y(5) = 336,\ \ y(7) = 720.$ Hence obtain $y(8)$.

28) The following data gives the melting point of an alloy of lead and zinc; θ is the temperature in degree centigrade; x is the percent of lead. Find θ when $x = 84$.

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

29) From the following table, find the value of $e^{1.17}$ by using Gauss forward formula.

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
e^x	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

30) The following values of x and y are given . Find the value of $y(0.543)$.

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$y(x)$	2.631	3.328	4.097	4.944	5.875	6.896	8.013

31) Use Gauss interpolation formula to find y_{41} with help of following data

$y_{30} = 3678.2,\ \ y_{35} = 2995.1,\ \ y_{40} = 2400.1,\ \ y_{45} = 1876.2,\ \ y_{50} = 1416.2$

32) By using central difference formula find the value of log 337.5 satisfying the following table

x	310	320	330	340	350	360
$log x$	2.4014	2.5052	2.5185	2.5315	2.5441	2.5563

33) Values of $y = \sqrt{x}$ are listed in the following table, which are rounded off to 5 decimal places. Find $\sqrt{1.12}$ by using Stirling's formula

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
$y = \sqrt{x}$	1.00000	1.02470	1.04881	1.07238	1.09544	1.11803	1.14017

34) By using Lagrange's formula, express the following rational fraction as sum of partial fractions $\frac{x^2+6x+1}{(x^2-1)(x^2-10x+24)}$.

35) By means of Lagrange's formula prove that approximately

$$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}\left[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})\right]$$

36) Apply Lagrange's formula to find the root of $f(x) = 0$ when
$f(30) = -30,\ f(34) = -13,\ f(38) = 3, f(42) = 18.$

37) Use Stirling's formula to find u_{32} for the following table
$u_{20} = 14.035,\ u_{25} = 13.674,\ u_{30} = 13.257,\ u_{35} = 12.734,\ u_{40} = 12.089,$
$u_{45} = 11.309.$

38) Construct the divided difference table for the given data and evaluate $f(1)$.

x	-4	-2	-1	0	2	5	10
$f(x)$	469	47	7	1	-5	271	7091

39) Use Newton's divided difference interpolation to obtain a polynomial $f(x)$ satisfying the following data of values and hence find$f(5.)$

x	-1	0	3	6	7
$f(x)$	3	-6	39	822	1611

40) Prove that the third order divided difference of the function $f(x) = \frac{1}{x}$ with arguments a, b, c, d is $-\frac{1}{abcd}$.

UNIT-III

41) Fit a straight line of the form $y = a + bx$ to the data

x	0	5	10	15	20	25	30
y	10	14	19	25	31	36	39

42) Find best values of a, b, c so that the parabola $y = a + bx + cx^2$ fits the data

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.2	1.5	2.6	2.8	3.3	4.1

43) Fit a second degree parabola of the $y = ax^2 + bx + c$ to the following data.

x	0	1	2	3	4
y	1	5	10	22	38

44) Determine the constants a and b by the method of least squares such that $y = ae^{bx}$ fits the following data:

x	2	4	6	8	10
y	4.077	11.084	30.128	81.897	222.62

45) Fit a function of the form $y = ax^b$ to the following data:

x	2	4	7	10	20	40	60	80
y	43	25	18	13	8	5	3	2

46) Find the values of a_0 and a_1 so that $y = a_0 + a_1 x$ fits the data given in the table

x	0	1	2	3	4
y	1.0	2.9	4.8	6.7	8.6

47) Find $\frac{d}{dx}(J_0)$ at $x = 0.1$ from the data given in the table:

x	0.0	0.1	0.2	0.3	0.4
$J_0(x)$	1.0000	0.9975	0.9900	0.9776	0.9604

48) Find the first and second derivatives of $f(x)$ at the point $x = 3.0$ from the following table:

x	3.0	3.2	3.4	3.6	3.8	4.0
$f(x)$	-14.000	-10.032	-5.296	0.256	6.672	14.000

49) From the following table of values of x and y obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 2.2$

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3210	4.0552	4.9530	6.0496	7.3891	9.0250

50) The following table of values of x and y is given :

x	0	1	2	3	4	5	6
y	6.9897	7.4036	7.7815	8.1291	8.4510	8.7506	9.0309

Find $\frac{dy}{dx}$ $at\ x = 3.$

51) From the following values of x and y, find $\frac{dy}{dx}$ when $x = 6$.

x	4.5	5.0	5.5	6.0	6.5	7.0	7.5
y	9.69	12.90	16.71	21.18	26.37	32.34	39.15

52) Find the minimum and maximum values of the functions from the following table

x	0	1	2	3	4	5
$f(x)$	0	0.25	0	2.25	16.00	56.25

53) Evaluate by using Trapezoidal rule

a) $\int_0^{\pi} tsint\ dt$ (with 6 strips)

b) $\int_{-2}^{2} \frac{t}{5+2t} dt$ (with 8 strips)

54) When a train is moving at 30 miles an hour, steam is burnt off and breaks are applied. The speed of the train in miles per hour after t seconds is given by :

t	0	5	10	15	20	25	30	35	40
v	30	24	19.5	16	13.6	11.7	10.8	8.5	7.0

Determine how far the train has moved in 40 seconds.

55) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{sin\theta}\ d\theta$, using Simpson's rule with h=$\frac{\pi}{12}$.

56) Use the Simpson's $\frac{3^{th}}{8}$ rule to obtain an approximation of $\int_0^{0.3} (1 - 8x^3)^{\frac{1}{2}} dx$ with h=0.05.

57) Evaluate $\int_0^1 cosxdx$ using h=0.2.

58) Find the value of $\int_3^7 x^2\ logx\ dx$ by taking 8 strips using Boole's rule.

59) Use Weedle's rule to obtain an approximate value of π from the formula $\int_0^1 \frac{1}{1+x^2} dx = \frac{\pi}{4.}$

60) Apply Trapezoidal and Simpson's rules to the integral $I = \int_0^1 \sqrt{1 - x^2} dx$ by dividing the range into 10 equal parts.

UNIT-IV

61) Use Matrix inversion method to solve the system of equation:
$3x + 2y + 4z = 7,\ \ 2x + y + z = 7,\ x + 3y + 5z = 2.$

62) Use Matrix inversion method to solve the system of equation:
$x + 2y + 3z = 10,\ 2x - 3y + z = 1,\ \ \ 3x + y - 2z = 9.$

63) Solve the following system of equations using Gauss elimination method:
$x_1 - 2x_2 - x_4 = 2,\ \ 2x_1 + 2x_2 + x_3 + 2x_4 = 7$
$3x_1 - x_2 - 2x_3 - x_4 = 3,\ \ x_1 - 2x_4 = 0.$

64) Solve the following system of equations using Gauss elimination method:
$2x_1 + x_2 + 4x_3 = 12,\ 8x_1 - 3x_2 + 2x_3 = 20,$
$4x_1 + 11x_2 - x_3 = 33$

65) Solve the following system of equations using Factorization method:
$5x - 2y + z = 4,\ 7x + y - 5z = 8,\ 3x + 7y + 4z = 10.$

66) Solve the following system of equations using Factorization method:
$2x - 3y + 10z = 3, -x + 4y + 2z = 20, 5x + 2y + z = -12.$

67) Solve the following system of equations using Jacobi's iterative method:
$10x + 2y + z = 9,\ 2x + 20y - 2z = -44,\ -2x + 3y + 10z = 22.$

68) Apply Gauss-siedal iterative method to solve:
$10x + y + z = 12,\ 2x + 10y + z = 13,\ \ 2x + 2y + 10z = 14.$

69) Apply Gauss-siedal iterative method to solve:
$27x + 6y - z = 85,\ \ 6x + 15y + 2z = 72,\ \ x + y + 54z = 110.$

70) Solve the following system of equations using Jacobi's itterative method:
$17x_1 + 65x_2 - 13x_3 + 50x_4 = 84,\ 12x_1 + 16x_2 + 37x_3 + 18x_4 = 25$
$56x_1 + 23x_2 + 11x_3 - 19x_4 = 36,\ 3x_1 - 5x_2 + 47x_3 + 10x_4 = 18.$

71) Using Taylor's series method to find the value of $y(0.1)$ and $y(0.2)$ if $y(x)$ satisfies $\frac{dy}{dx} = x - y^2$ with $y(0) = 1$.

72) Solve $\frac{dy}{dx} = x + y$ by Taylor's series method starting with $x_0 = 1, y_0 = 0$ and carry to $x = 1.2$ with $h = 0.1$. Compare the final result with the value of explicit solution.

73) Using Picard's method solve $\frac{dy}{dx} = 1 + xy$ with $y(0) = 1$. Find $y(0.1), y(0.2) \dots y(0.5)$.

74) Use Picard's method to approximate y upto 3 decimal places when $x = 0.2$.Given that $y(0) = 1$ and $\frac{dy}{dx} = x - y$.

75) Using Euler's method, solve the following initial value problems:

i) $\frac{dy}{dx} + 2y = 0, \quad y(0) = 1$

ii) $\frac{dy}{dx} - 1 = y^2, \ y(0) = 0$ in each case take $h = 0.1$ and obtain $y(0.1), \ y(0.2), \ y(0.3)$.

76) Given $\frac{dy}{dx} = x^2 + y, \ y(0) = 1$ determine $y(0.02), \ y(0.04), \ y(0.06)$ using modified Euler's method

77) Find y when $x = 0.1, \ x = 0.2, \ x = 0.3$ from the following initial value problem by Runge-Kutta's 4th order method $y' = x - y^2, \quad y(0) = 1$

78) Given $\frac{dy}{dx} = 1 + y^2$, where $y = 0$ when $x = 0$, find$y(0.2), \ y(0.4), \ y(0.6)$ by Runge-Kutta's 4th order method.

79) Apply Milne's method to the equation $y' = x + y^2$ with $y(0) = 0$ to find $y(0.8)$. (take $h = 0.2$ to obtain initial values)

80) Using Milen's method solve the differential equation $(1 + x)\frac{dy}{dx} + y = 0$ with $y(0) = 2$. Find $y(1.5)$. (take $h = 0.5$ to obtain initial values)